By Zied Ben Chaouch

15.401 Managerial Finance

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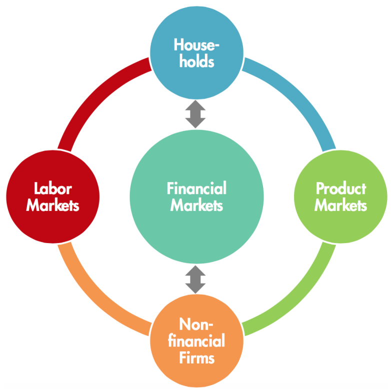
Introduction



Key Function of Financial Products

Invest - Transfer money across time periods:

- Deposit accounts, retirement, or pension funds
- Stocks, government and corporate bonds
- Mortgages

Insure - Protect the value of an asset:

- Insurance, options, forwards & futures, credit default swaps

What is Finance

Valuing a Company - How much is a business venture worth ? Present Value, Capital Budgeting

Raising Capital - Ways to finance a business venture & use them ? Bonds & Stocks, Cap. Structure, Real Estate Fin., Financing a Startup

Managing Risk - How to measure risk & hedge it ? Diversification, Risk & Returns, Options

Principles of Finance

Principles Value of an **Asset** = Value of its **Cash Flows**.

More\$ > less\$ - Investors Prefer More to Less

Today\$ > Tomorrow\$ - Money paid in the future is worth less than the same amount today

Safe\$ > Risky\$ - Investors are risk averse

No Arbitrage - Financial markets are competitive

Definition (Arbitrage) Transaction where profit is made with no additional risk by simultaneously buying and selling an asset that is not properly priced in two markets.
Buying smth at one price, sell it at other price, without risk, when cost of doing the transaction < difference in price.

Note: Many assets traded on markets (stocks, bonds, commodities, futures, options, ...). Can use market prices to extract information:

- The prices reflect the general consensus
- Every time new information is released, the market prices adjust

Net Present Value

Time Value of Money

CFs are discounted for two reasons:

(1) 1\$ today is worth more than 1\$ tomorrow.
(2) A safe 1\$ is worth more than a risky 1\$.

Present Value (PV) - How much is a cash flow at time t (CF_t) worth today ($PV(CF_t)$) using discount rate r ?

$$PV(CF_t) = \frac{CF_t}{(1+r)^t}$$

Net Present Value (NPV) - Value entire stream of cash flows $(CF_0, CF_1, \cdots, CF_T)$?

$$NPV(CF_0, CF_1, \cdots, CF_T) = CF_0 + \frac{CF_1}{(1+r)} + \cdots + \frac{CF_T}{(1+r)^T}$$

Theorem A project is good $\iff NPV > 0$

Assumptions CFs known with certainty (amount & time) + Discount Rate known & constant.

Properties X_t, Y_t cash flows, $a \in \mathbb{R}$:
$NPV(a \cdot X_0, a \cdot X_1, ..., a \cdot X_T) = a \cdot NPV(X_0, X_1, ..., X_T)$
$NPV(X_0 + Y_0, ..., X_T + Y_T) = NPV(X_0, ..., X_T) + NPV(Y_0, ..., Y_T)$
$NPV(X_0, ..., X_T) = NPV(X_0, ..., X_\tau) + NPV(X_{\tau+1}, ..., X_T)$

Discount Rate r Determined by rates of return prevailing in capital markets.

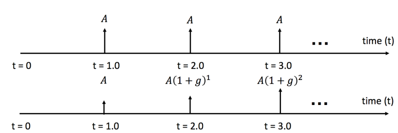
- Safe futureCF?r ≈interest rate on safe securities(US treasury bonds)
- Uncertain CF?r ≈rate of return of equivalent-risk securities (stocks)

Warning: Small changes Δr can lead to big changes ΔPV.

Future Value (FV) - How much is a cash flow today (PV_{CF}) worth at time t (FV_t) at rate of return r ?

$$FV_t = PV_{CF} \cdot (1+r)^t$$

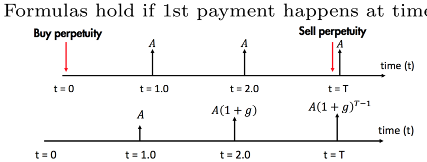
Special Cash Flows



Definition (Perpetuity) A stream of fixed payments per period for an **infinite number of periods**.

Proposition $CF_t = A$ for all $t \geq 1$:
$PV(Perpetuity) = \frac{A}{r}$
$PV(Perpetuity) = \frac{A}{r-g}$ if constant growth g

Warning: Formulas hold if 1st payment happens at time $t = 1$.



Definition (Annuity) A stream of fixed payments per period for a **specific number of periods**.

Proposition $CF_t = A$ for all $1 \leq t \leq T$:
$PV(Annuity) = \frac{A}{r}\left(1 - \frac{1}{(1+r)^T}\right)$ (buy perp at $t = 0$, sell at $t = T$)

$PV(Annuity) = \frac{A}{r-g}\left(1 - \frac{(1+g)^T}{(1+r)^T}\right)$ if constant growth $g < r$

$PV(Annuity) = T \cdot \frac{A}{1+r}$ if constant growth $g = r$

Warning: Formulas hold if 1st payment happens at time $t = 1$.

Compounding

Discount rates vs. Interest rates:

Discount Rate :
- Number r that makes you indifferent between 1\$ today and $(1+r)$\$ after one unit of time.
- Subjective number obtained from considering time preference and opportunity cost.
- Typically used for valuation: given a stream of cash flows, can calculate net present value.

Interest Rate :
- Given a "face value" or price of an asset (stream of CFs), interest rate = a number r that makes NPV equation hold. - Can also go backwards: use r to compute face value or payment.

Link :
- Interest rates representative of average "market" discount rates. - Your personal discount rate may also be affected by interest rate through opportunity cost of investment.

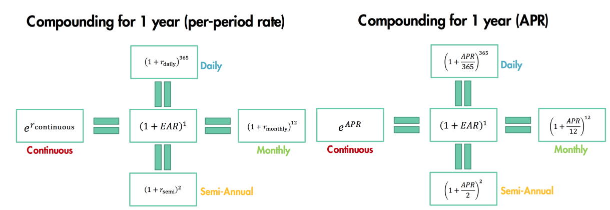
Definition (Annual Percentage Rate, APR) :
APR = Interest rate per period (e.g., month) × # of periods in a year.

Definition (Effective Annual Rate, EAR) :
$(1 + EAR) = (1 + APR)^{\#\text{ of periods in a year}} = (1 + APY)$.
Also called **Annual Percentage Yield (APY)**

Example: Invest 1\$ at APR = r **compounded m times per year**:
- Investment at end of year worth $= \left(1 + \frac{r}{m}\right)^m \times 1\$$.
- Effective interest rate (EAR/APY) $= \left(1 + \frac{r}{m}\right)^m - 1$.

Example: Compound $1,000\$$ at 10% APR for 30 years.
- Compounded once every 30 years: $4,000\$$.
- Compounded once a decade: $8,000\$$.
- Compounded yearly: $17,449\$$.
- Compounded daily: $20,077\$$.

Proposition $N = \#$ of periods in a year.
$EAR = \left(1 + \frac{APR}{N}\right)^N - 1$
$APR = N\left[(1 + EAR)^{\frac{1}{N}} - 1\right]$
Compounding Period < 1 year $\implies$ EAR > APR
Low interest rates + not too much compounding $\implies APR \approx EAR$



Note: If interest is paid evenly throughout the year: interest often quoted as a **continuously compounded rate**.

Inflation: Nominal vs. Real

"Nominal" rates of return are the prevailing/quoted market rates. Nominal CFs are expressed in **dollars at each date**.

"Real" rates are nominal rates adjusted for inflation. Real CFs are expressed in **constant purchasing power**.

Usual problem phrased in terms of nominal rates + nominal cash flows. Beware if asks you to account for inflation or for the real rate.

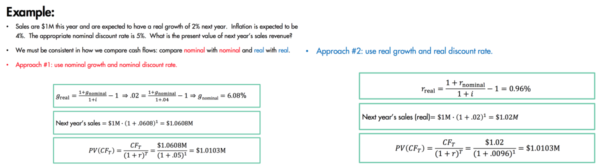
Be consistent − if CFs nominal apply nominal discount rate; if CFs real apply real rate.

Example: (Nominal vs. real cash flows) Inflation is 4% per year. You expect to receive 1.04\$ in one year, what is this CF really worth next year? The inflation adjusted, or real value of 1.04\$ in a year, where i is the inflation rate:
$(\text{Real CF})_t = \frac{(\text{Nominal CF})_t}{(1+i)^t} = \frac{1.04\$}{1+0.04} = 1\$$

Definition (Inflation Risk) When cash flows are measured in nominal terms (dollars), real cash flows are exposed to inflation risk (risky even if delivery is certain). Exposure to inflation risk depends on financial position: Lenders lose/Borrowers gain.

Example: (Nominal vs. real rates of return) :
−Nominal rates of return are the prevailing market interest rates
−Real rates of return are the inflation adjusted rates.
1.00\$ invested at a 6% interest rate grows to 1.06\$ next year.
However, if inflation is 4% per year, then the real rate of return is:
$r_{real} = \frac{1+r_{nominal}}{1+i} - 1 = \frac{1+0.06}{1+0.04} - 1 = 1.9\%$
$g_{real} = \frac{1+g_{nominal}}{1+i} - 1$

Note: $r_{real} \approx r_{nominal} - i$



Capital Budgeting

Using NPV

Theorem (NPV Rule) Project with CFs $\{CF_0, CF_1, \cdots, CF_T\}$:
Current Market Value: $NPV = CF_0 + \frac{CF_1}{1+r} + ... + \frac{CF_T}{(1+r)^T}$.
$\Longrightarrow$ Only take projects with positive NPV.

- Single project: take $\iff$ is NPV positive.
- Many independent projects: take all those with positive NPV.
- Mutually exclusive proj: take one with positive & highest NPV.

When computing NPV:

1. Use CFs, not accounting earnings
2. Use after-tax CFs
3. Use cash flows attributable to the project:
   - Use incremental CFs
   - Forget sunk costs: bygones are bygones (money spent in past and not recoverable should be ignored)
   - Include investment in working cap and in cap expenditure
   - Include opportunity costs of using existing facilities
   - Be consistent in treatment of inflation

Definition (Capital Expenditures, CapEx) Funds used by a company to acquire or upgrade physical assets such as property, industrial buildings or equipment. It can include everything from repairing a roof to building, to purchasing a piece of equipment, or building a brand new factory.

Accounting: Depreciate CapEx linearly over multiple periods. Depreciate **more**: smaller loss/looks better; **less**: larger CFs. **Warning:** Depreciation matters for CFs through taxes!

CFs Do NOT include depreciation (not a CF), but include CapExp
CF = Project cash Inflows − Outflows
CF = (Operating Revenues) − (Non CapExp w/o Depr)− CapExp −(Income Tax)

Non CapExp = COGS + OpExp

- **Cost of Goods Sold:** COGS = direct costs attributable to the production of the goods sold by a company. This amount includes the cost of the materials used in creating the good along with the direct labor costs used to produce the good.
- **Operating Expenses:** An expense incurred in carrying out an organization's day-to-day activities, but not directly associated with production. Operating expenses include such things as payroll, sales commissions, employee benefits and pension contributions, transportation and travel, amortization and depreciation, rent, repairs, and taxes.

Operating Profit/Income
= (Operating Revenues) − (Non CapExp w/o Depr−(Depr)

- **Taxes** Income taxes = $\tau\times$ Op.Prof.
- **Operating Revenues:** what I get from sales
  Op.Rev = $\tau\times$ Op.Prof.

Definition (EBITDA) Earnings Before Interests, Taxes, Depreciation, and Amortization (=depr for intangible assets: patents...)
Measures how much cash is coming in now.
EBIT: Earnings Before Interests & Taxes
Definition (Working Capital WC) = Assets − Liab.
WCap = Inventory + Acc.Receivable − Acc.Payable
Measures company's liquidity, efficiency, and overall health:
WC = money available to a company for day-to-day operations.

- **Inventory**: Raw materials, work-in-process products + finished goods considered ready for sale (or soon).
  COGS ignores cost of items produced but not sold.
  Inventory ↑: COGS understates cash outflows;
  Inventory ↓: COGS overstates cash outflows.
- **Accounts Receivable**: Money owed to the firm for sales by its clients/customers. Accounting sales may reflect sales that have not been paid for. Accounting sales understate cash inflows if the company is receiving payment for sales in past periods.
- **Accounts Payable**: reverse of Accounts Receivable.
- **Change in WC, ΔWC** Measures delays between recorded sale and cash received.
  $\Delta WC > 0$ (< 0) : company (UN)able to pay off its short-term liabilities almost immediately. Equal ↑ in CF.
  $\Delta WC < 0$ suggests a company is becoming over-leveraged, struggling to maintain or grow sales, paying bills too quickly, or collecting receivables too slowly (opposite if $\Delta WC > 0$). Equal ↓ in CF.

- $CF = (1-\tau) \times EBITDA - CapEx + \tau \times Depr. - \Delta WC + [Salv.Val - \tau \times (Salv. - BookVal)]$
- $CF = (1-\tau) \times EBIT - CapEx + Depr. - \Delta WC + [Salv.Val - \tau \times (Salv. - BookVal)]$

Other Capital Budgeting Methods

Good Tools? Can give same answer as NPV, but in general they do not. **Use NPV whenever possible !**

Why Firms Use Them? Used historically + may have worked (in combination with common sense) in their past.

NPV issues:
Include competitive response of other companies. Capital rationing (fixed budget).

Possible sources of positive NPV: Short-run competitive advantage (right place at the right time) + Long-run competitive advantage (patent, technology, economies of scale, etc.).
⇒ Always take into account competitor's response + study cause of positive NPV.

Definition (Internal Rate of Return, IRR) IRR = discount rate that makes the projects NPV equal to zero:
$CF_0 + \frac{CF_1}{(1+IRR)} + \frac{CF_2}{(1+IRR)^2} + \ldots + \frac{CF_k}{(1+IRR)^k} \overset{!}{=} 0.$

Proposition (IRR Decision Rules)

- Independent projects: accept a project if its $IRR > IRR^*$ (some fixed "threshold rate")
- Mutually exclusive projects: among the projects having $IRR > IRR^*$, accept one with the highest IRR

Proposition (IRR ∼ NPV) IRR leads to same decision as NPV if

- Cash outflow occurs only at time 0
- Only 1 project is under consideration
- Opportunity cost of capital is the same for all periods
- Threshold rate IRR^* is set equal to opportunity cost of capital

Proposition (Shortcomings of IRR)

- IRR may not exist (no solution of eq)
- IRR may not be unique (no unique solution of eq)
- IRR does not take the project size into account
- IRR does not take into account different time patterns
  Fix for last two: use incremental CFs

Definition (Payback Period) time to recoup amount invested.
Payback period = minimum k such that:
$CF_1 + \cdots + CF_k \geq -CF_0 =: I_0$.
(minimum length of time such that sum of cash flows from a project is positive)

Proposition (Payback Decision Rules)

- Independent projects: accept a project if its $k \leq t^*$ (some fixed "threshold rate")
- Mutually exclusive projects: among the projects having $k \leq t^*$, accept one with the lowest k

Proposition (Shortcomings of Payback Method)

- Ignores time-value of money
- Ignores cash flows after k

Definition (Discounted Payback Method) time to recoup amount invested. Minimum k such that:
$\frac{CF_1}{(1+r)} + \cdots + \frac{CF_k}{(1+r)^k} \geq -CF_0 =: I_0$.
Warning: Still ignores cash flows after k.

Definition (Profitability Index) Ratio of the PV of future CFs and the initial cost of a project:
$PI := \frac{PV}{-CF_0} = \frac{PV}{I_0}$.
(minimum length of time such that sum of cash flows from a project is positive)

Proposition (PI Decision Rules)

- Independent projects: **accept** a project if its $PI \geq 1$ ($\sim$ NPV rule)
- Mutually exclusive projects: among the projects having $PI \geq 1$, **accept** one with the highest PI

Proposition (PI $\sim$ NPV) PI leads to same decision as NPV if

- Cash outflow occurs only at time 0
- Only 1 project is under consideration

Proposition (Shortcomings of PI) PI scales projects by their initial investments. The scaling can lead to wrong answers in comparing mutually exclusive projects.
Fix: use incremental CFs

Bonds

Bond Markets

Definition (Bonds/Fixed-Income Securities:) Financial claims with promised CFs of fixed amount paid at fixed dates.
Future CFs are fixed $\Rightarrow$ the price of the security today changes ($\because$)interest rates change and the PV of those future cash flows changes
A few types of fixed-income securities:

Treasury securities:

- US Treasury securities: bills (short term $<$ 6Mo), notes (medium term $5-10$y), bonds (long term: $10-30$y)
- Bunds, JGBs (Japanese Govt Bonds), UK Gilts...

Federal Agency Securities: Securities issued by federal agencies e.g., FHLB (Federal Home Loan Banks),... ex: Mortgage backed securities $-$ Fannie Mae, Freddy Mac

Corporate securities: Commercial paper (CP), Medium-term notes (MTNs), Corporate bonds

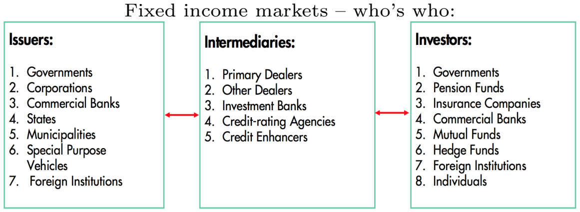
Municipal securities: Munis

Mortgage-backed securities: MBS

Asset backed securities: ABS

Note: Before 2007: MBS drive the growth of outstanding U.S. bond market debt. After crisis, rapid expansion of treasury debt.

Fixed income markets – who's who:



Definition (Central Bank) (US: Federal Reserve) plays a critical role in fixed-income markets.
Sets interest rates through monetary policy (**Open Market Operations, Discount Window, Interest on Reserves**).
Since the financial crisis, dominant investor in fixed income assets.
Quantitative Easing: QE1, QE3 $-$ Mortgage-Backed Securities. QE2 $-$Treasury Securities

Proposition (Bond Properties) CFs depends on:

- Maturity of bond: time of bond's final payout
- Principal/Face Value/Par Value: Full value of bond (excluding interest)
- Coupon: periodic interest payments
- Price of a bond = PV of remaining CFs

Warning: At bond's maturity date: pay last coupon + principal.

Term Structure Of Interest Rates:
The term structure of interest rates defines interest rates for investments of different time horizons
Idea: The time value of money is described by **spot interest rates** or equivalently, the prices of **discount bonds** (e.g. **zero-coupon bonds**). By ignoring other risks (default risk, liquidity) + focusing solely on the time value of money, we can calculate the discounted value of riskless cash flows. Additional risks can be added later.
Definition (Spot Rates) Spot interest rate r_t is the current (**annualized**) interest rate for a maturity date of t.
Note: r_t = rate for payments in t periods from now. r_t different for each different maturity t.

Example:
Spot rates for U.S. Treasury securities in % (03/01/2017):

1 Mo	3 Mo	6 Mo	1 Yr	2 Yr	3 Yr	5 Yr	10 Yr	20 Yr	30 Yr
0.46	0.63	0.79	0.92	1.29	1.57	1.99	2.29	2.46	2.81

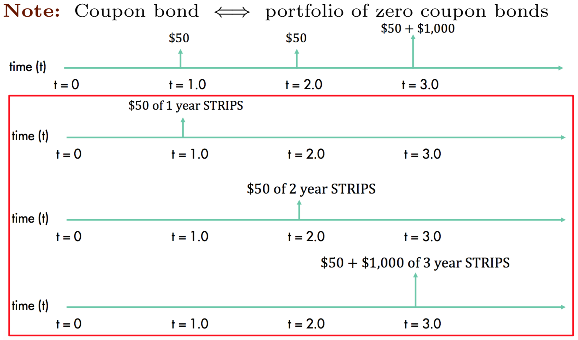
If I put 1$ now, I get $(1.0157)^3$$ in 3 yrs.
If $r_t < 0$, negative interest rates: investors keep suitcases of money or spend cash.

Definition (Term Structure of Interest Rates) Set of spot interest rates for different maturities.

Definition (Discount Bond/Zero-Coupon Bond) Bond which pays $ only at maturity t: CF_t = Face Value of Bond
$PV(CF_t) = \frac{CF_t}{(1+r_t)^t}$ and $r_t = \left(\frac{CF_t}{PV}\right)^{1/t} - 1$ is spot rate
Example:
STRIPS (Separate Trading of Registered Interest and Principal Securities)

Maturity	3 Mo.	6 Mo.	1 Yr.	2 Yr.	5 Yr.	10 Yr.	30 Yr.
Price	0.9999	0.9997	0.9981	0.9898	0.9127	0.7383	0.2938

Definition (Coupon Bond) Bond pays stream of regular coupon payments + a principal payment at maturity t.
$CF_k = (\text{Coupon Rate} + \mathbb{I}_t(k)) \times$ Face Value of Bond
Note: Coupon bond $\Longleftrightarrow$ portfolio of zero coupon bonds



Proposition (Price of Coupon Bond)

- Given discount bond prices B_t:
  ex: 3-Years 1000$ Bond, 5% coupon rate:
  Bond Price $= 50B_1 + 50B_2 + (1000+50)B_3 = 998.65$$

t	1	2	3	4	5
B_t	0.952	0.898	0.863	0.807	0.757

- Given Spot Rates:
  $B = \frac{C_1}{(1+r_1)^1} + \ldots + \frac{C_{t-1}}{(1+r_{t-1})^{t-1}} + \frac{P+C_t}{(1+r_t)^t}$ where B = Bond Price, C = Coupon and P = Principal.
- Given **Yield To Maturity (YTM)**:
  $B = \frac{C_1}{(1+y)^1} + \ldots + \frac{C_{t-1}}{(1+y)^{t-1}} + \frac{P+C_t}{(1+y)^t}$
  YTM replaces the different spot rates + solves for one rate y given the same price + stream of cash flows + maturity. ex: Spot rates $r_1 = 5\%$, $r_2 = 6\%$, price of 2 yr bond with face value $P = 100$$ and coupon rate 6% is: $B = \frac{6}{(1+0.05)^1} + \frac{106}{(1+0.06)^2} = 100.0539$ and YTM $= 5.9706\%$ (solve for y in $100.0539 = \frac{6}{(1+y)^1} + \frac{106}{(1+y)^2}$)
  Note: YTM close to 6% since most CF in year 2

Hypothesis on Interest Rates:
Idea: Term structure of interest rates determined by: Expected future spot rates + Market's consensus for risk associated with long bonds.
Definition (Yield Curve) Shape of term structure of interest rates. Various models of interest rates try to quantify different drivers that shape the yield curve. ex: Expectation Hypothesis, Liquidity preference, Dynamic models (Vasicek, Cox-Ingersoll-Ross)
Proposition (Expectation Hypothesis) Long-term interest rates are related to current and expected short-term interest rates.
Let $r_{t,m}$ = rate on maturity m at date t. Then:
$(1+r_{t,m})^m = (1+r_{t,1})(1+\mathbb{E}_t[r_{t+1,1}])\ldots(1+\mathbb{E}_t[r_{t+m-1,1}])$
Warning: Only a hypothesis! There are other differences between short-term and long-term bonds! (liquidity is important)
Proposition (Liquidity Preference Hypothesis)
$(1+r_{t,2})^2 = (1+r_{t,1})(1+\mathbb{E}_t[r_{t+1,1}]) +$ Liquidity Premium
Note: **Implications:**

- On average, long term bonds receive higher returns than short term bonds.
- Term structure reflects:
  Expectations of future interest rates + Risk premium demanded by investors in long term bonds because they are less liquid

Fixed-Income Risk

Interest Rate Risk:
Average Rate of Return on US Treasuries (1952-2016)

Avg Inflation: 3.43%	$<$ 6Mo:	$5-10$Yr:
Nominal	4.72%	6.26%
Real	1.29%	2.83%

Definition (Interest Rate Risk)
Interest rates change stochastically over time $\Rightarrow$ bond prices change $\Rightarrow$ value of bond subject to interest rate risk.
Note: The more periods you compound, the more Δr affects ΔB, so: Longer Bonds $\Rightarrow$ Larger Risks
Returns from holding a bond can be highly volatile, even when the payoff is certain! Volatility is higher the longer you need to wait for your payments.

Duration, a measure of interest rate risk exposure:
Assume: flat term structure at $r_t = y$
Definition (Macaulay Duration) weighted avg term to maturity.
$D = \sum_{t=1}^{T} \frac{PV(CF_t)}{B} \times t = \frac{1}{B}\sum_{t=1}^{T} \frac{CF_t}{(1+y)^t} \times t$,
where bond price $B = \sum_{t=1}^{T} \frac{CF_t}{(1+y)^t}$
Definition (Modified Duration) Relative price change with respect to a unit change in yield
$\Rightarrow$ measures a bond's interest rate risk
$MD = -\frac{1}{B}\frac{\Delta B}{\Delta y} = \frac{D}{1+y}$
$\Rightarrow \Delta B\% = \frac{\Delta B}{B} \approx -MD \cdot \Delta y$ yield $\Uparrow$ price $\Downarrow$
Proposition

- D = weighted avg time it will take to get your payments
- MD = measures how sensitive the price of a bond ΔB is to market interest rates Δr. ex: $MD = 8$, so $r \Uparrow 1\%$ (e.g., $5-6\%$) $\Rightarrow$ $B \Downarrow 8\%$
- D(portfolio) = weighted average of D(constituents)
  $D_P = \sum_x \frac{PV_x}{PV_P} \times D_x$ for Portfolio P of bonds x
- D(Zero Coupon Bond) = maturity m
- $D \leq$ maturity always, and
  $D \leq$ maturity of zero-coupon bond of same duration

- Coupon rate $\Uparrow$ and all else equal $\Rightarrow$ $D \Downarrow$
- YTM $\Uparrow$ and all else equal $\Rightarrow$ $D \Downarrow$ $(\because)$ discount future more
- $D(\text{perpetuity}) = \frac{1+y}{y}$ (perpetual debt of yield y)
- Immediately after a coupon payment, D_{bond} decreases ? NO!
- Term structure of interest rates $\Uparrow \Rightarrow$ investors expect higher short term interest rates in future ? No/Uncertain

Definition (Portfolio Immunization/Duration Matching)
Immunize your portfolio against interest rate changes: make the duration of assets and liabilities equal $\Rightarrow$ interest rate changes makes the values of assets and liabilities change by the same amount
$MD_{assets} \times PV_{assets} = MD_{liab} \times PV_{liab}$
$(\because) P = \text{Price: } \Delta P \approx -\frac{D_{assets}}{1+y} P_{assets} \Delta y + \frac{D_{liab}}{1+y} P_{liab} \Delta y \overset{!}{=} 0$

Price risk: interest rate $\Downarrow \Rightarrow$ PV of the bonds $\Uparrow \Rightarrow$ PV of the liabilities $\Uparrow$ more!

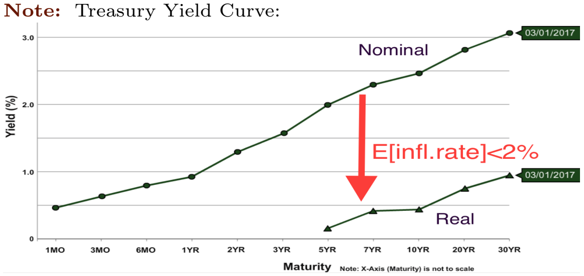
Reinvestment risk: At new interest rate, assets cannot be reinvested to make future payments

Example: (Immunize Your Portfolio) $x = \#$ of 1yr bonds with price P_1, $y = \#$ of 30yr bonds with price P_{30}, $PV(\text{Portfolio}) = P$.
$P = xP_1 + yP_{30}$ and $MD_P P \overset{!}{=} 1Yr \times \frac{x}{1+r} + 30Yr \times \frac{y}{(1+r)^{30}}$

Inflation Risk:
Most bonds give nominal payoffs: **inflation risk** $\Rightarrow$ risky real payoffs (even if nominal payoffs are safe).
ex: Nominal interest rate = 10%, Inflation rate = 10%, 8%, 6% w.p.1/3, Real interest rate = 2%, Year 0 value = 1,000\$.
Return from investing in a 1-year Treasury security:

Inflation Rate	Y1 Nominal Payoff	Y1 Real Payoff
10%	1,100\$	1,000\$
8%	1,100\$	1,019\$
6%	1,100\$	1,036\$

Note: Treasury Yield Curve:



Default Risk:
Excluding government bonds, other fixed-income securities (ex: corporate bonds) carry default risk.
Definition (Default/Credit Risk:)
Risk that a debt issuer fails to make the promised payments.
Bond ratings by rating agencies (Moody's, S&P, Finch) indicate the statistical likelihood of default by each issuer.

Description	Moody's	S&P	Comment
Gilt-edge	Aaa	AAA	Investment Grade
Very high grade	Aa	AA	Investment Grade
Upper medium grade	A	A	Investment Grade
Lower medium grade	Baa	BBB	Investment Grade
Low grade	Ba	BB	Speculative (Junk)

Note:
In recession times, spread between Aaa and Baa bond yields blows up.
Default rates on Moody's IG corporate bonds higher for 5Yr than 1Yr.
Corporate bonds can also be downgraded (and companies can default).

Capital Structure

How Do Firm Finance Its Projects

Equity residual claim on the assets of the company, implies an ownership claim on the company, and generally carries voting rights.

Debt loan with fixed terms (sometimes containing restrictive covenants on the operations of the company). Debt holders have prior claim on assets of the company, but no ownership or voting rights

Assets/Value Assets = Equity + Debt

Definition (Leverage Ratio) $LR = \frac{\text{Assets}}{\text{Equity}} = \frac{E+D}{E}$

Example: (Highly Leveraged Firm Risk) $LR = 33\%$

$$100^{\$}\ (A) = 3^{\$}\ (E) + 97^{\$}\ (D) \quad (1)$$
$$101^{\$}\ (A) = 4^{\$}\ (E) + 97^{\$}\ (D) \quad (2)$$
$$97^{\$}\ (A) = 0^{\$}\ (E) + 97^{\$}\ (D) \quad (2')$$

From (1) to (2), the firm gains 1\$: $(A) \uparrow 1\%$, $(E) \uparrow 33\%$.
From (1) to (2′), the firm looses 3\$: $(A) \downarrow 3\%$, $(E) \downarrow 100\%$.
Bank won't give a new loan because too risky (2nd loan is junior loan, so each \$ you loose is a direct loss for them). Also, new issued debt can have impact on market value of existing debt.

Definition (Sources of Cash) to finance new projects + ongoing activities of firm:

Internal Funds: Cash generated by operations

Equity: Issuing new stock: private equity vs. IPO

Debt: bank loans vs. issuing bonds

Alternatives: convertibles, options, other securities

$\Rightarrow$ CFO: Which sources of funds should we use ?
$\Rightarrow$ Way projects are financed determines ownership of future CFs & firm capital structure

Capital Structure

Definition (Capital Structure:)
How firm's assets & future projects are financed.
Represents mix of **rights** & **claims** to firm's Assets & CFs.
Characteristics of **Financial Claims**:

Payoff Structure: Fixed vs. variable payment

Priority/Seniority: Debt paid before Dividends. Senior debt first.

Maturity

Covenants: Restrictions, ex: corporate debt.

Voting Rights

Options

$\Rightarrow$ Optimal combination of Debt & Equity ? MM: NO!

Theorem (Modigliani-Miller, 1958) Assume that:

1. Complete markets: can create any portfolio you want
2. Efficient markets: rational investors/no information asymmetry
3. No taxes
4. No transaction or bankruptcy costs
5. Investment policy is consistent (firm behavior unaffected by Cap Structure)

$\Rightarrow$ The market value of any firm is independent of its capital structure: any combination of securities is as good as another. This means that a project's CF is NOT affected by the way it was financed: same NPV. Projects/firms prices are equated by arbitrage if their CFs are equal: pure financial transactions should NOT change the value of firms.

$$V = \sum_{t=1}^{\infty} \frac{CF_t}{(1+r)^t} = \sum_{t=1}^{\infty} \frac{CF_t - rD}{(1+r)^t} + \sum_{t=1}^{\infty} \frac{rD}{(1+r)^t} = E + D$$

Assume: no tax shield from debt + equal discount/interest rates $r_E = r_D$ (usually $r_E > r_D$ as E paid after D so riskier) + CFs do not depend on financing
Example: (MM) Project A: 100 million $1^{\$}$ shares issued.
Project B: 50 million $1^{\$}$ shares + 50 million $1^{\$}$ bonds issued.

Demand	Equity (A)	Bond (A)	Equity (B)	Bond (B)
High EPS	150 $M^{\$}$ 1.5\$	$0^{\$}$	100 $M^{\$}$ 1\$	50 $M^{\$}$
Lo EPS	50 $M^{\$}$ 1\$	$0^{\$}$	$0^{\$}$ 0\$	50 $M^{\$}$

MM intuition:

1. **"Pie theory":** same investor owns both E and D (e.g. bonds) $\Rightarrow$ gets same payoff no matter what! Completely indifferent to capital structure. The firm divides the pie among different claimants without changing its size.
   ex: Value(A) = E(A) $\overset{!}{=}$ Value (B) = E(B) + D(B)
2. **"Firm debt vs. shareholder debt":** Investors will not pay a premium for a firm to undertake financial transactions that they can undertake themselves. Doesn't care if he borrows or if firm borrows.
   ex: Either invest $200^{\$}$ in project B: return = $200^{\$}$ or $0^{\$}$ or borrow $50^{\$}$ and invest $(150+50)^{\$}$ in A: return = $200^{\$}$ or $0^{\$}$.
3. **"Market efficiency":** if securities sold at market prices, no loss of value.
   ex: Firm wants to raise $150M^{\$}$: (A) all E, (B) E+D. Under both plans firm keeps assets worth $150M^{\$}$, indifferent. Transactions are purely financial & create 0 NPV.

Weighted Average Cost of Capital (WACC)

All five are no longer considered valid $\rightarrow$ Debt/Equity balance matters. How do we know what is the right balance? WACC
$\Rightarrow$ Capital structure can define the feasibility of projects and affect cost of equity.
Definition (Rate of Return) = Payment/Price
r_V, r_D, r_E: Return to buying asset, collecting a payment, selling asset.
Definition (Leverage Ratio) $\lambda = \frac{\text{Debt}}{\text{Assets}} = \frac{D}{E+D}$
Assume a firm wants to invest using a constant leverage ratio: Debt D, Value of Assets V; Constant leverage $\Rightarrow D = \lambda V$
Interest Rate on Debt/Cost of Debt: $r_D = \frac{r_D D}{D}$
Cost of Equity: $r_E = \frac{(1-\tau)(CF - r_D D)}{E}$
Rate of Return of the Firm: $r_V = \lambda r_D + (1-\lambda) r_E$
$(\because) r_V = \frac{(1-\tau)CF + \tau r_D D)}{V} = \frac{r_D D}{V} + \frac{(1-\tau)(CF - r_D D)}{E} = \lambda r_D + (1-\lambda) r_E$

Note: Usually $r_E > r_D$ as r_E is riskier than r_D.
Debt changes $\Rightarrow r_D$ & r_E move.
Example:

$$1^{\$}\ \text{Today} \Rightarrow (1+r_V)^{\$}\ \text{tomorrow}$$
$$\lambda^{\$}\ \text{Debt} \Rightarrow \lambda(1+r_D)^{\$}$$
$$(1-\lambda)^{\$}\ \text{Equity} \Rightarrow (1-\lambda)(1+r_E)^{\$}$$

Note: Shareholders are indifferent to increase in leverage ratio: $\lambda \uparrow \Rightarrow \mathbb{E}[\text{returns}] \uparrow \Rightarrow$ Risk $\uparrow$

Proposition If CFs are constant (i.e., $CF_t = CF\ \forall t$), the **Total Firm Value** is: $V = \frac{(1-\tau)CF}{(1-\tau)\lambda r_D + (1-\lambda) r_E} \stackrel{!}{=} \frac{(1-\tau)CF}{r_{WACC}}$

Proof: $V = \sum_{t=1}^{\infty} \frac{(1-\tau)CF + \tau r_D D}{(1+r_V)^t} = \frac{(1-\tau)CF + \tau r_D D}{r_V}$.
Use $D = \lambda V$ and $r_V = \lambda r_D + (1-\lambda) r_E$. $\square$

Definition (Weighted Average Cost of Capital, WACC)
Discount rate used to evaluate projects for a company.
How much should the firm make on 1\$ to be able to repay debts r_D and meet the required rate of return r_E for investors ?
Lower WACC is better!

$$\begin{aligned} r_{WACC} &= (1-\tau)\lambda r_D + (1-\lambda) r_E \\ &= (1-\tau) r_D \frac{D}{D+E} + r_E \frac{E}{D+E} \end{aligned}$$

Proposition (General WACC Formula)

$$V = \sum_{t=1}^{\infty} \frac{(1-\tau)CF_t}{(1+r_{WACC})^t} \quad \text{, and} \quad E = V - D$$

Note: Value of firm $\uparrow \Rightarrow$ WACC $\downarrow$

Remarks:

- Discount rates are project-specific. Each project like stand-alone firm.
- Leverage ratios should be the project's operational target
- Cost of equity is difficult to observe.
  - Estimated with similar "pure play" firms (with similar capital structure) or the parent firm's cost of capital if it is comparable.
- Cost of debt could be estimated with current market rate charged to comparable firms with similar credit risk
  - If the project has different layers of debt, an average cost of debt should be estimated
- Tax rate should be that of the firm undertaking the project
  - Average tax rate is not useful.
  - Need marginal tax rate (on additional 1\$ of profit). May be difficult to observe in practice.
- If the project capital structure is expected to remain stable, then WACC will be stable. Otherwise, WACC should change.
  - In practice, firms tend to use a stable WACC regardless.
- Optimal capital structure: choose λ to maximize NPV.
  - Why not all debt ? interest rate r_D would increase.

Recap:

- Modigliani-Miller: benchmark where capital structure doesn't matter (perfect markets, no tax benefits, no distress costs...).
- In practice: major tax benefit from debt.
- Key tool: weighted average cost of capital (WACC).
- Value of firm: $V = \frac{(1-\tau)CF - \tau r_D D}{r_V}$ with $r_V = \lambda r_D + (1-\lambda) r_E$
- Usually target leverage ratio $\lambda = D/V$ but difficult to compute as is
- Instead use: $V = \frac{(1-\tau)CF}{r_{WACC}}$ with $r_{WACC} = \lambda(1-\tau) r_D + (1-\lambda) r_E$
- All NPV accrues to equity holders.

Stocks

Stock Market

Definition (Common Stocks)
Represent equity or ownership positions in a corporation
+ give right to payments made in various forms:
Cash dividends, Stock dividends, Share repurchases.
$\Rightarrow$ payments are uncertain in timing + magnitude: depend on firm's performance & policy.
Legal Characteristics:

Residual claim to assets and cash flows after creditors.

Limited liability stockholders may lose only invested amount.

Voting rights to elect Board of Directors (BOD) and major corporation decisions at shareholders meetings or by proxy.

Definition (Preferred Stocks) Generally no voting rights but prior claim on earnings and assets. Dividends paid first to preferred stocks.

Definition (Stock Market) Organized trading of securities through physical and electronic exchanges and over-the-counter (OTC)

Definition (Primary Market) Underwriting securities

Venture capital: A company issues shares to investment partnerships, investment institutions + wealthy individuals

Initial public offering (IPO): A company issues shares to general public for the first time (i.e., "going public")

Secondary (seasoned) offerings (SEO): A public company issues additional shares (r_E usually high, don't like it much)

Definition (Secondary Market) Exchanges and OTC

Exchanges: NYSE, NASDAQ, ...

OTC: dark pools (anonymous exchanges: don't know how much a buyer/seller is offering, o'wise seller/buyer sees you're desperate and charge you a higher price). **Note:** why trade on exchange rather than dark pools ? can't afford to wait

Definition (Trading in secondary market:) **Trading costs** commission, bid-ask spread, price impact (unless in dark pool, if make a trade you affect price)

Buy on margin Someone gives you money that you invest.

Long: Buying of a security such as a stock, commodity or currency with the expectation the asset will rise in value

Short Position/Short Selling: Sell security (you don't own) at price P_0 at $t=0$, and buy it at price P_1 at $t=1$: profit$= P_0 - P_1$. Short selling is motivated by the belief that a security's price will decline, enabling it to be bought back at a lower price to make a profit. **Note:** Risky! P_1 unbounded, so loss can be infinite
Note: Annual equity returns can vary a lot from year to year (ex: 1931: -44.4%, 1933: $+57.5\%$; 2008: -38.3%, 2009: 26.4%). However stocks are kind of safe in the long run.

Common Stocks Valuation Models

Definition (Discounted Dividend Model) Basic PV formula applied to stock valuation given info on: Expected future dividends + Future dividend risk (discount rate)

$$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+r_t)^t},$$

where P_t = Stock price at time t ex-dividend, D_t = expected cash dividend at t, r_t = risk-adjusted discount rate for cash flow at t.

Definition (Gordon Growth) Constant Growth Discounted Divid Model: (r constant, D_t grow in perpetuity at constant rate $g < r$)

$$P_0 = \sum_{t=1}^{\infty} \frac{(1+g)^t D_1}{(1+r)^t} = \frac{D_1}{r-g} = \frac{D_0 \times (1+g)}{r-g}$$

Definition (Dividend Yield) $= \frac{D_0}{P_0}$ or $\frac{D_1}{P_0}$

Definition (Implied Cost of Capital) $r = \frac{D_1}{P_0} + g = \frac{D_0}{P_0}(1+g) + g$
where g = growth rate of dividends in long run.
$\Rightarrow$ Cost of Capital = Div.Yield + Div.Growth !

Key variables to forecast dividends:

Earnings E = Profits $-$ Depr. = Dividends + New investments - Depr.

Earnings (per share): EPS = total profit net of depr. and taxes

Payout ratio: p = Dividends/Earnings = $\frac{DPS}{EPS}$

Retained earnings: RE =Earnings$-$Dividends

Plowback ratio: $b = 1 - p = RE$/Earnings

Book value: BV = Cumulative RE

Return on book equity: ROE = Total Earnings/BV

Note: Based on accounting data, NOT market values!
b: is the amount you put back into the firm.
BV: when you buy something it's worth what you bought it for

Proposition

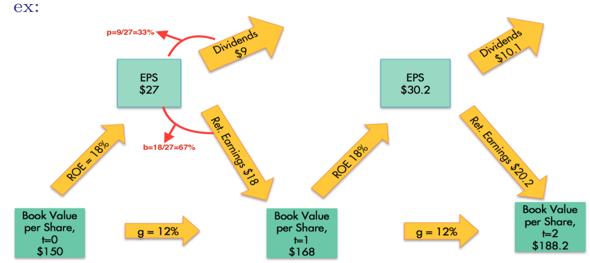
$$\Delta BV = \text{RE}$$

Proof: $\text{BV}_{t+1} = \text{BV}_t$ + New Investments $-$ Depr., so
ΔBV = New Investments $-$ Depr. But RE = EarningsDividends
= Dividends + New investments $-$ Depr.Dividends
= New investments $-$ Depr. $= \Delta BV$ $\square$

Proposition Useful Identities:

- $EPS_1 = BVS_0 \times ROE$, with BVS = BV per Share
- $BVS_1 = BVS_0 \times (1+g)$
- $b = \frac{RES_1}{EPS_1}$, with RES = RE per Share
- Growth Rate of BV: $\frac{\Delta BV}{BV} = \frac{RE}{BV} = \frac{RE}{Earnings} \times \frac{Earnings}{BV} = b \times ROE$
- $Earnings = BV \times ROE$ and $RE = b \times BV \times ROE$
- $Dividends = Earnings - RE = (1-b) \times BV \times ROE = p \times BV \times ROE$

Note: b and ROE constant $\Rightarrow$ Dividends & BV must grow at the same rate (($\because$)their ratio to remain constant)
ex:



Note: If $ROE = r$: growth plan irrelevant, price will be the same under all scenarios.
If $ROE > r$: $g \Uparrow$
If $ROE < r$: $g \to 0$
If ROE and growth g constant: $EPS_1 = ROE \times BV_0$, $RES_1 = g \times BV_0$, $D_1 = (ROE - g)BV_0$, and $P_0 = \frac{(ROE-g)BV_0}{r-g}$

Definition (Multi-Stage Growth)

Growth Stage: Rapidly expanding sales, high profit margins, and high growth in earnings per share, many new investment opportunities, low dividend payout ratio (($\because$)reinvest in company).

Transition Stage: Growth rate and profit margin reduced by competition, fewer new investment opportunities, high payout ratio (($\because$)reinvesting a lot in company no longer makes sense).

Mature Stage Earnings growth, payout ratio and average return on equity stabilizes for the remaining life of the firm.

Growth Opportunities

Definition (Growth Opportunities) Investment opportunities that earn expected returns higher than the cost of capital ($ROE > r$)

Definition (Growth Stocks, GS) Stocks of companies that have access to growth opportunities.
NOT necessarily GS: a stock with growing EPS, Dividends or Assets.
Maybe GS: stock with EPS(or DPS)growing slower than cost of capital

Definition (PVGO) Stock price has 2 components: PV of
-Earnings under a no-growth policy: $P_0 = \frac{EPS_1}{r}$
-Growth opportunities (PVGO): $P_0 = \frac{EPS_1}{r} + PVGO$
Note: $D_1 = p \times EPS_1$ and $g = b \times ROE$

Proposition $PVGO < 0 \Leftrightarrow ROE < r$:firm should distribute $ as div.

Some Terminology:

Earning Yield $\frac{EPS_1}{P_0}$

Price-Earning Ratio $\frac{P_0}{EPS_1}$ (higher for growth stocks)

Note: Reported P/E ratio is typically $\frac{P_0}{EPS_0}$. But we care about future earnings.
Which have higher returns on average, growth stocks or non-growth (“value”) stocks?

Definition (Pai Mei)

- Legendary master of Bak Mei + Eagle’s Claw styles of kung fu.
- Lived for over 1000 years + poisoned in 2003.
- Likes to eat fish heads.

Book Stuff

CFs

By now present value calculations should be a matter of routine. However, forecasting project cash flows will never be routine. Here is a checklist that will help you to avoid mistakes:

1. Discount cash flows, not profits.
    - Remember that depreciation is not a cash flow (though it may affect tax payments).
    - Concentrate on cash flows after taxes. Stay alert for differences between tax depreciation and depreciation used in reports to shareholders.
    - Exclude debt interest or the cost of repaying a loan from the project cash flows. This enables you to separate the investment from the financing decision.
    - Remember the investment in working capital. As sales increase, the firm may need to make additional investments in working capital, and as the project comes to an end, it will recover those investments.
    - Beware of allocated overhead charges for heat, light, and so on. These may not reflect the incremental costs of the project.
2. Estimate the project’s incremental cash flows that is, the difference between the cash flows with the project and those without the project.
    - Include all indirect effects of the project, such as its impact on the sales of the firm’s other products.
    - Forget sunk costs.
    - Include opportunity costs, such as the value of land that you would otherwise sell.
3. Treat inflation consistently.
    - If cash flows are forecasted in nominal terms, use a nominal discount rate.
    - Discount real cash flows at a real rate.
4. Separate investment and financing decisions by forecasting CFs as if the project is all equity financed.

Modigliani-Miller

- Modigliani and Millers (MMs) famous proposition 1 states that no combination is better than any other – that the firms overall market value (the value of all its securities) is independent of capital structure.
- Firms that borrow do offer investors a more complex menu of securities, but investors yawn in response. The menu is redundant. Any shift in capital structure can be duplicated or “undone” by investors. Why should they pay extra for borrowing indirectly (by holding shares in a levered firm) when they can borrow just as easily and cheaply on their own accounts?
- MM agree that borrowing raises the expected rate of return on shareholders’ investments. But it also increases the risk of the firm’s shares. MM show that the higher risk exactly offsets the increase in expected return, leaving stockholders no better or worse off.
- Proposition 1 is an extremely general result. It applies not just to the debt-equity trade- off but to any choice of financing instruments. For example, MM would say that the choice between long-term and short-term debt has no effect on firm value.
- The formal proofs of proposition 1 all depend on the assumption of perfect capital markets. MM’s opponents, the “traditionalists”, argue that market imperfections make personal borrowing excessively costly, risky, and inconvenient for some investors. This creates a natural clientele willing to pay a premium for shares of levered firms. The traditionalists say that firms should borrow to realize the premium.
- Proposition 1 is violated when financial managers find an untapped demand and satisfy it by issuing something new and different. The argument between MM and the traditionalists finally boils down to whether this is difficult or easy. We lean toward MM’s view: Finding unsatisfied clienteles and designing exotic securities to meet their needs is a game that’s fun to play but hard to win.
- If MM are right, the overall cost of capital – the expected rate of return on a portfolio of all the firm’s outstanding securities – is the same regardless of the mix of securities issued to finance the firm. The overall cost of capital is usually called the company cost of capital or the weighted-average cost of capital (WACC). MM say that WACC doesn’t depend on capital structure. But MM assume away lots of complications. The first complication is taxes. When we recognize that debt interest is tax-deductible, and compute WACC with the after-tax interest rate, WACC declines as the debt ratio increases. There is more – lots more – on taxes and other complications in the next two chapters.

Diversification

Portfolio Characteristics

Randomness:

$\mathbf{P_0} =$ Price at the beginning of the period.

$\widetilde{\mathbf{P_1}} =$ Price at the end of the period **(very uncertain)**.

$\widetilde{\mathbf{D_1}} =$ Dividend at the end of the period **(uncertain)**.

Note: Dividends less risky than price as next quarter's dividends depends mainly on this time period, while price aggregates info and predictions about future of the firm.

$\widetilde{\mathbf{r_1}} = \frac{\widetilde{D_1}+\widetilde{P_1}}{P_0} - 1 =$ Return.

$\mathbf{E}[\widetilde{\mathbf{r_1}}] =$ Expected Return.

$\widetilde{\mathbf{r_1}} - \mathbf{r_F} =$ Excess Return.

$\mathbf{E}[\widetilde{\mathbf{r_1}}] - \mathbf{r_F} =$ Risk Premium.

Useful Statistics:

Mean: $\overline{r_1} = E[\tilde{r_1}] = \sum_i p_i r_i$. Estimator: $\hat{r} = \frac{1}{T}\sum_{t=1}^T r_t$.

Variance: $\sigma^2 = \text{Var}(\tilde{r_1}) = E[(\tilde{r_1} - \overline{r})^2] = \sum_i p_i (r_i - \overline{r})^2$.
Estimator: $\widehat{\sigma^2} = \frac{1}{T-1}\sum_{t=1}^T (r_t - \hat{r})^2$.

Standard Deviation: $\sigma = \sqrt{\sigma^2}$. Estimator: $\hat{\sigma} = \sqrt{\widehat{\sigma^2}}$.

Median: 50^{th} percentile, value s.t. half of observations are below the median.

Skewness: is the distribution symmetric ? Negative: big losses more likely than big gains ($\sim$ stock market); Positive: big gains more likely than big losses.

Kurtosis: does the distribution have fat tails ? $\kappa = E\left[\left(\frac{\tilde{r}-\overline{r}}{\sigma}\right)^4\right]$

Covariance:
$\sigma_{ij} = \text{Cov}(\tilde{r_i}, \tilde{r_j}) = E[(\tilde{r_i} - \overline{r_i})(\tilde{r_j} - \overline{r_j})] = \sum_i p_i (r_i^a - \overline{r^a})(r_i^b - \overline{r^b})$.
If $\sigma_{ij} > 0$: both variables go up or down together; If $\sigma_{ij} < 0$: both variables go in opposite directions.

Correlation: $\rho_{ij} = \text{Corr}(\tilde{r_i}, \tilde{r_j}) = \frac{\text{Cov}(\tilde{r_i}, \tilde{r_j})}{\sigma_i \sigma_j}$ ranges from -1 to 1.

Beta: $\beta_{ij} = \frac{\text{Cov}(\tilde{r_i}, \tilde{r_j})}{\sigma_j^2}$. When $\tilde{r_j}$ goes up by 1, $\tilde{r_i}$ goes up by β_{ij} on average.

Portfolio:

Portfolio: Combination of different assets where each of them can be defined by its **Number of Shares** N_i and **Share Price** P_i.

Total Value of a Portfolio: $V = \sum_i N_i P_i$

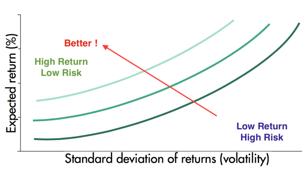
Portfolio Weights: $w_i = \frac{N_i P_i}{N_1 P_1 + \ldots + N_n P_n}$, with $w_1 + \ldots + w_n = 1$. If $w_i > 0$: **long** position; If $w_i < 0$: **short** position.

Q: Why not pick the best asset instead of forming a portfolio?

- Dont know which stock is best
- Portfolios reduces unnecessary risks
- Portfolios can enhance performance by focusing bets
- Portfolios can customize and manage risk/reward trade-offs

Q: How do we chose the "best" portfolio? What characteristics of a portfolio do we care about?

- Risk and reward(expected return)
- Like: Higher expected returns/ Don't like: Higher risks



Definition (Indifference curve:) A set of return and volatility $(E[\tilde{r}], \sigma)$ combinations that give an investor the same expected utility.

Mean-Variance Utility: $U(\tilde{r}) = \mathbb{E}[\tilde{r}] - \frac{1}{2} \cdot A \cdot \text{Var}[\tilde{r}]$

Parameter $A > 0$: measures **risk aversion** ($A = 0$: risk neutral, cares only about expected return, not about risk).

$\Rightarrow$ Investor indifferent between getting return U for sure and gambling on return $\tilde{r}$.

How To Select A Desirable Portfolio::

Investor preferences:

1. Investors prefer more to less
2. Investors like high expected returns but dislike high volatility
3. Investors care about the performance of their overall portfolio (Not individual stocks/investors are well diversified)

Optimal portfolio reconciles what is desirable (indifference/ utility curves) with what is feasible (efficient frontier)

Portfolio Characteristics:

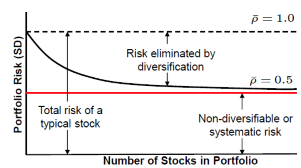
$$\begin{aligned} r_p &= w_1 r_1 + \ldots + w_n r_n \\ \mathbb{E}[\tilde{r_p}] &= w_1 \mathbb{E}[\tilde{r_1}] + \ldots + w_n \mathbb{E}[\tilde{r_n}] \\ \text{Var}[\tilde{r_p}] &= \mathbb{E}\left[(\tilde{r_p} - \overline{r_p})^2\right] = \sum_{i=1}^n \sum_{j=1}^n w_i w_j \text{Cov}[\tilde{r_i}, \tilde{r_j}] \end{aligned}$$

To compute covariance, add up all of entries in:

	$w_1 r_1$	$w_2 r_2$	$\ldots$	$w_n r_n$
$w_1 r_1$	$w_1^2 \sigma_1^2$	$w_1 w_2 \sigma_{12}$	$\ldots$	$w_1 w_n \sigma_{1n}$
$w_2 r_2$	$w_2 w_1 \sigma_{21}$	$w_2^2 \sigma_2^2$	$\ldots$	$w_2 w_n \sigma_{2n}$
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
$w_n r_n$	$w_n w_1 \sigma_{n1}$	$w_n w_2 \sigma_{n2}$	$\ldots$	$w_n^2 \sigma_n^2$

Diversification reduces risks as long as correlation$\neq 1$, can get higher returns with lower risks ($\sigma_p < \sigma_i \,\forall i$). If $\rho_{ij} = -1$, stocks (i,j) move in completely opposite directions to each other $\Rightarrow$ perfect diversification.

Definition (Non-Diversifiable risk) (or market/ systematic/ common risk) comes from Business cycle, Inflation, Volatility, Credit, Liquidity,...



Proposition (Diversify to ∞) Portfolio with n assets **equally weighted**:

$$\begin{aligned} \sigma_p^2 &= \frac{1}{n}\left(\frac{1}{n}\sum_{i=1}^n \sigma_i^2\right) + \frac{n^2 - n}{n^2}\left(\frac{1}{n^2 - n}\sum_{i=1}^n \sum_{j \neq i}^n \sigma_{ij}\right) \\ &= \underbrace{\frac{1}{n}(\text{avg variance})}_{\longrightarrow 0} + \underbrace{\frac{n^2 - n}{n^2}(\text{avg covariance})}_{\longrightarrow \text{avg covariance}} \end{aligned}$$

Optimal Portfolios

Definition (Lowest Risk (1)/Highest Return (2) Approach) **Minimize** the total risk/volatility (1) or **Maximize** the expected return (2) of the portfolio **subject to**:

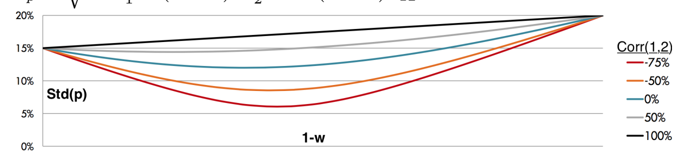
- Having a balanced portfolio
- Achieving the desired level of return (1) or risk (2)

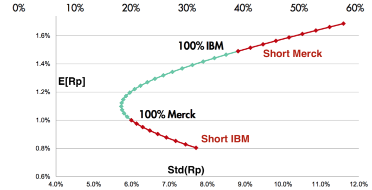
$$\min_{\{w_1,\ldots,w_n\}} \quad \sigma_p^2 = \sum_{i,j=1}^n w_i w_j \sigma_{ij}^2 \qquad \min_{\{w_1,\ldots,w_n\}} \quad \mathbb{E}[r_p] = \sum_{i=1}^n w_i \mathbb{E}[r_i]$$

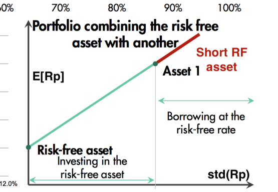
$$\text{Subject To} \quad \sum_{i=1}^n w_i = 1 \qquad \text{Subject To} \quad \sum_{i=1}^n w_i = 1$$

$$\sum_{i=1}^n w_i \mathbb{E}[r_i] = \mathbb{E}[r_p] \qquad \sum_{i,j=1}^n w_i w_j \sigma_{ij}^2 = \sigma_p^2$$

Example: Two assets: $\sigma_1 = 15\%$ (weight w), $\sigma_2 = 20\%$ ($w_2 = 1 - w$).
$\sigma_p = \sqrt{w^2\sigma_1^2 + (1-w)^2\sigma_2^2 + 2w(1-w)\sigma_{12}}$





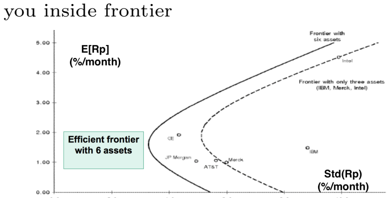


Definition (Risk-Free Asset) Characteristics of the risk-free asset:

- Return = risk free rate (U.S. treasury bond of same duration)
- $\sigma_{rf} = 0$, $\text{Cov}(r_{rf}, r_i) = 0$, $\forall i$

Theorem Include more assets $\Rightarrow$ portfolio frontier improves: (moves toward upper-left: higher mean returns & lower risk)
($\because$)can always choose to ignore the new assets, so including them cannot make you worse off.

- Range of feasible solutions forms an area
- **Mean-Variance Frontier Portfolio:** portfolio that minimizes risk (measured by the standard deviation/variance), given an expected return
- **Portfolio Frontier:** locus of all frontier portfolios in the mean-standard deviation plane
- **Efficient Frontier:** upper part of the portfolio frontier. Combination of risky assets with the best risk-return profile
- Convex curve ($\because$)mixing 2 portfolios (convex combination) keeps you inside frontier

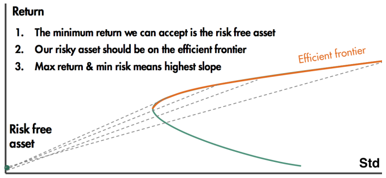


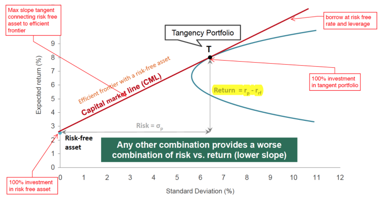
Definition (Tangency Portfolio) Best risky asset combination.
$\Rightarrow$ Has highest possible Sharpe Ratio

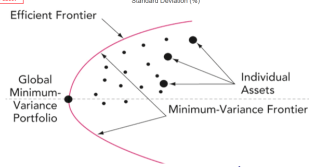
Definition (Capital Market Line, CML) Combination of the tangency portfolio and the risk-free asset.
$\Rightarrow$ All CML portfolios have the best possible Sharpe ratio: highest expected return for given risk

Definition (Sharpe Ratio) Sharpe Ratio $= \frac{\mathbb{E}[\tilde{r_p}] - r_{RF}}{\sigma_p}$

$\Rightarrow$ Select portfolio with best Sharpe ratio & desired risk







Example: (Minimum variance portfolio) For two assets, to find the minimum variance portfolio: set the derivative of the portfolio's variance to zero. Weight: $w_1 = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}$, with $\sigma_{12} = \rho_{12}\sigma_1\sigma_2$.

Recap:

- Portfolio risk depends primarily on covariance (not on stocks' individual volatilities).
- Diversification can reduce some, but not all, risk (Idiosyncratic/ stock-specific risk but not Systematic/ common risk)
- The CML has the lowest risk for a given expected return, and the highest expected return for a given level of risk. All the portfolios on the CML have the highest possible Sharpe Ratio.
- People are risk averse: they should hold portfolios on the efficient frontier or the CML.

Risks and Return

Portfolio Theory

Perfect negative correlation is implausible in real life: Diversification does work, but most companies earnings are positively correlated with each other.
Idiosyncratic risk requires no "reward" as it can be diversified: free lunch if it did have a reward.
Systematic risk requires "reward", otherwise no one would be willing to be exposed to it.
Idea: Which discount rate r should we use ? If risk is completely diversifiable $\Rightarrow$ use risk-free rate r_{rf}

Market Portfolio:
Assumptions Investors agree on the expected return of assets.

- Investors hold efficient frontier portfolios.
- There is a risk free asset: pays interest rate r_{rf} + Investors can borrow at this rate as well as save
- In equilibrium, demand equals supply of assets

Then:

- Every investor puts his money into two pots: the riskless asset + a single portfolio of risky assets: the Tangency Portfolio
- All investors hold the risky assets in same proportions: The weights in the Tangency Portfolio
- **If all investors hold Tangency Portfolio, then Tangency Portfolio = Market Portfolio**

Example: The market has 3 risky assets A, B, and C. Tangency portfolio: $w_A = 25\%$, $w_B = 50\%$, $w_C = 25\%$. Three investors with wealth of \$500, \$1,000, \$1,500. Their asset holdings (in billions \$) are:

Investor	Riskless	A	B	C
1	$100^{\$}$	$100^{\$}$	$200^{\$}$	$100^{\$}$
2	$200^{\$}$	$200^{\$}$	$400^{\$}$	$200^{\$}$
3	$-300^{\$}$	$450^{\$}$	$900^{\$}$	$450^{\$}$
Total	0\$	$750^{\$}$	$1,500^{\$}$	$750^{\$}$
Market Portfolio	0%	25%	50%	25%

$\Rightarrow$ Market Portfolio = Tangency Portfolio

Capital Asset Pricing Model (CAPM)

Assumptions

- Investors like portfolios with high $\mathbb{E}[r_p]$, dislike portfolios with high σ_p, and don't care about anything else:rational+risk averse
- No asymmetric information: investors have same estimate of $E[r_i]$ and $\text{Cov}(r_i, r_j)$ $\forall$ risky assets
- $\exists$ a risk-free asset for both borrowing and investing.

Definition (Capital Asset Pricing Model, CAPM) Efficient portfolios are combinations of market portfolio and T-Bills. The expected return of an asset i must satisfy:

$$\mathbb{E}[r_i] = r_{rf} + \beta_i \times (\mathbb{E}[r_M] - r_{rf})$$

with $\beta_i = \frac{\text{Cov}(r_i, r_M)}{\text{Var}(r_M)} = \rho_{i,M}\frac{\sigma_i}{\sigma_M}$, r_M = Return of Market Portfolio.
For an arbitrary portfolio p:

$$\mathbb{E}[r_p] = r_{rf} + \beta_p \times (\mathbb{E}[r_M] - r_{rf})$$

with $\beta_p = w_1\beta_1 + \ldots + w_n\beta_n$. **Note:** $SR(P) = \beta_p \frac{\sigma_m}{\sigma_p} SR(M)$

Corollary If CAPM holds, then an asset's expected return depends on how it comoves with the market (β_i):

- $\beta_i = 1 \Rightarrow \mathbb{E}[r_i] = \mathbb{E}[r_m]$
- $\beta_i = 0 \Rightarrow \mathbb{E}[r_i] = r_{rf}$: only get compensated for systematic risk
- $\beta_i < 0 \Rightarrow \mathbb{E}[r_i] < r_{rf}$: need higher price (holding stock $\downarrow$ risk)

Understanding the CAPM:
Define: $r_{i,t+1} - r_{f,t} = \alpha_i + \beta_i(r_{M,t+1} - r_{f,t}) + \epsilon_{i,t+1}$
$\Longrightarrow$ $\mathbb{E}[\epsilon_{i,t+1}] = 0$, and $\text{Cov}(\epsilon_{i,t+1}, rM, t+1) = 0$
$\Longrightarrow$ $\beta_i = \beta_{im}$ measure i's systematic risk
$\Longrightarrow$ $\text{Var}(\epsilon_{i,t+1})$ measure i's idiosyncratic risk
$\Longrightarrow$ α_i measure i's return beyond its risk adjusted award (by CAPM)
$\Longrightarrow$ $\underbrace{\text{Var}(r_{i,t+1})}_{\text{total risk}} = \underbrace{\beta_i^2 \text{Var}(r_{M,t+1})}_{\text{systematic risk}} + \underbrace{\text{Var}(\epsilon_{i,t+1})}_{\text{idiosyncratic risk}}$

The α parameter:

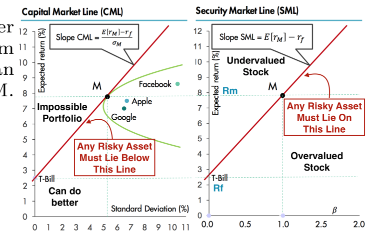
- CAPM $\Longrightarrow$ no free reward $\Longrightarrow$ $\alpha_i = 0$
- If $\alpha_i > 0$: improve Sharpe Ratio by selling r_f bonds & buying i (can't be true in equilibrium: can't buy if nobody wants to sell)
- If you get $\alpha_i > 0$:
  - check estimation errors.
  - Past value of α_i may not predict future values (CAPM theory is about expectations of future returns)
  - $\alpha_i > 0$ may be compensated for other risks.

CAPM Implications: Investors should only be rewarded for syst risk + $\alpha = 0$ (β alone should explain all differences in $\mathbb{E}[r]$ among assets).

Definition (Risk Premium) $= E[r_M] - r_{rf}$ = reward for systematic risk ($\sim 7\%$ in U.S. historically). The CAPM assumes that the cost of capital of a project is given by the expected return of an efficient portfolio with the same systematic risk.

Definition (Capital & Security Market Line, CML & SML) Efficient Portfolios are combinations of risk-free assets (T-Bills) & the Market Portfolio. They do not have idiosyncratic risk.
CML = set of efficient portfolios

Note: Stock below SML is over valued: a smaller risk premium is paid for a particular beta than would be expected from the CAPM.



Where does the CAPM come from?:
You currently hold the market portfolio and want to increase the weight on stock i by a small amount:

$$\begin{aligned} r_p &\approx (1 - \Delta w_i) r_m + \Delta w_i r_i \\ \mathbb{E}[r_p] - \mathbb{E}[r_m] &\approx \Delta w_i \cdot (\mathbb{E}[r_i] - \mathbb{E}[r_m]) \\ \frac{\text{Var}(r_p) - \text{Var}(r_m)}{\text{Var}(r_m)} &\approx 2\Delta w_i \cdot (\beta_i - 1) \end{aligned}$$

$\Rightarrow$ if adds risk ($\beta_i > 0$): need more compensation than on old portfolio
$\Rightarrow$ if reduces risk ($\beta_i < 0$): need less comp. than on old portfolio

CAPM implementation:

$\mathbf{r_f}$ use T-Bill rates of similar duration

$\mathbf{r_m}$ use historical returns and future expectations (U.S.: $\sim 6-8\%$)

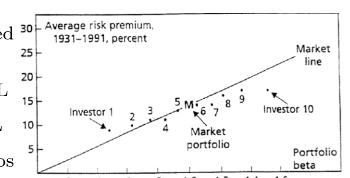
$\beta_i = \frac{\text{Cov}(r_i, r_m)}{\text{Var}(r_m)}$ OR lin. regress. $\begin{cases} r_i - r_f = \alpha_i + \beta_i(r_m - r_f) + \epsilon \\ r_i = \alpha_i + \beta_i \cdot r_m + \epsilon \end{cases}$

Note: Use Index as a proxy of the market (e.g., S&P 500)
+ Decide time horizon & frequency for the regression: use longest possible time horizon (with no structural break) & daily/ weekly freq
+ Regression: asset return ("y") vs. market return ("x"): β =slope

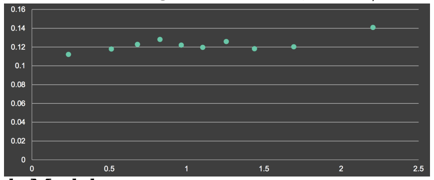
Debt and Equity Betas:
Firms are financed by debt and equity.
Definition (Asset Beta) β_A of the firm is the weighted average of the **debt beta** (β_D) and **equity beta** (β_E): $\beta_A = \frac{D}{D+E}\beta_D + \frac{E}{D+E}\beta_E$.
similar firms must have similar **asset** beta, but equity beta depends on capital structure

Limits of the CAPM:
Previous research shows that long-average returns are significantly related to beta.
Fisher Black investigated the historical returns for portfolios with different betas during the period 1931 through 1991 and he discovered the following:

- High beta portfolios generated higher average returns
- High beta portfolios below SML
- Low beta portfolios above SML
- A line fitted to the 10 portfolios would be flatter than SML

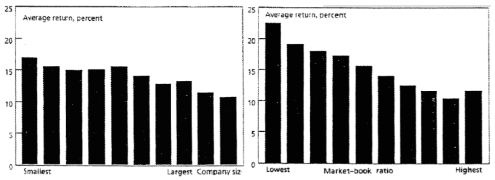


Updated evidence is less supportive... Value-weighted portfolios of U.S. equity returns ("y") vs portfolio betas: (Monthly returns (annualized), 60-month rolling window betas, NYSE/AMEX/NASDAQ



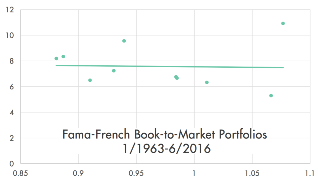
Fama French Models:
Beta not enough for asset pricing. Fama and French:

- Small stocks outperform large stocks
- Stocks with low market/book value ratio outperform stocks with high ratios



Book/MV decile	Avg. annual return	α
10^{th} decile	16.46%	4.95%
1^{st} decile	10.01%	−1.68%

Proposition CAPM dead: $\nexists$ relationship between beta & returns! ⇒ **∃ more sources of systematic risk (exposure to market)**
-Small stocks outperform large
-Market-to-Book-Value ratio: low ratio stocks outperform large ratio



Theorem (Fama-French 3-Factor Model) Add 2 new sources of systematic risk to the CAPM: return on a portfolio that goes

Size factor r_{smb}: long small stocks and short big stocks

Value factor r_{hml}: long high B/M stocks and short low B/M stocks

Augmented SML (three-factor model):
$\mathbb{E}[r_i] - r_f = \beta_{im}\left(\mathbb{E}[r_m] - r_f\right) + \beta_{is}\mathbb{E}[r_{smb}] + \beta_{ih}\mathbb{E}[r_{hml}]$

⇒ $\alpha = 0$ + superior fit: fraction of variance explained (R^2) goes from 25% to 75%

Arbitrage Pricing Theory (APT):

Theorem (APT) Extend the CAPM to include multiple sources of aggregate risks:

$$r_i - r_f = \alpha_i + \beta_{i1}\left(r_1^* - r_f\right) + \ldots + \beta_{in}\left(r_n^* - r_f\right) + \epsilon_i \quad \text{, where}$$

- $r_1, \ldots, r_n$ = common risk factors (e.g., r_m)
- $\beta_{i1}, \ldots, \beta_{in}$ = define asset i's exposure to each risk factor
- ϵ_i = part of asset is risk unrelated to risk factors

$$\mathbb{E}[r_i] - r_f = \beta_{i1}\left(\mathbb{E}[r_1^*] - r_f\right) + \ldots + \beta_{in}\left(\mathbb{E}[r_n^*] - r_f\right) \quad \text{, where}$$

- $\mathbb{E}[r_k^*] - r_f$ = premium on factor k
- $\beta_{i1}, \ldots, \beta_{in}$ = asset i's loading on factor k

Need to: Identify the factors & estimate factor loadings + premiums of assets

Note: APT gives a reasonable description of return and risk, but the model does not say what the right factors are.

Differences between CAPM and APT:

APT: based on the factor model of returns and "arbitrage"

CAPM: based on investors' portfolio demand and equilibrium

Options:

Definitions:

Options = bet that stocks goes up or down. Mostly to hedge unwanted risk: bet on interest rates & foreign exchange

Definition (Option Types)

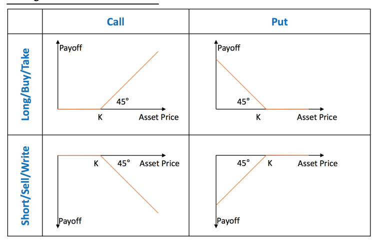
- **Call:** Holder has right to **buy** an asset at the strike price on/by time T: bet that asset's stock price will go **up** on or by T
- **Put:** Holder has right to **sell** an asset at the strike price on/by time T: bet that asset's stock price will go **down** on or by T

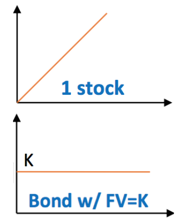
Definition (Exercise Style)

- **European:** owner can exercise the option AT time T
- **American:** owner can exercise the option on or before time T

Expiration/Maturity Date T. **Strike/Exercise Price** K. **Price of Underlying Asset** S_t at time T.

Payoff Profiles:





If Buy a European Call (at price C) or Put (at price P):

Option	Payoff	Profit
Call (buy)	$\max(S_T - K, 0)$	$\max(S_T - K, 0) - C(1+r)^T$
Call (sell)	$-\max(S_T - K, 0)$	$-\max(S_T - K, 0) + C(1+r)^T$
Put (buy)	$\max(K - S_T, 0)$	$\max(K - S_T, 0) - P(1+r)^T$
Put (sell)	$-\max(K - S_T, 0)$	$-\max(K - S_T, 0) + P(1+r)^T$

r = interest rate (EAR or (1+APR/n))

Note: Selling a put: potential loss may be infinite!

Definition ("In The Money") Option worth exercising: you will profit. The strike price K of **Call** (**Put**) option is **below** (**above**) the market price of the underlying asset.

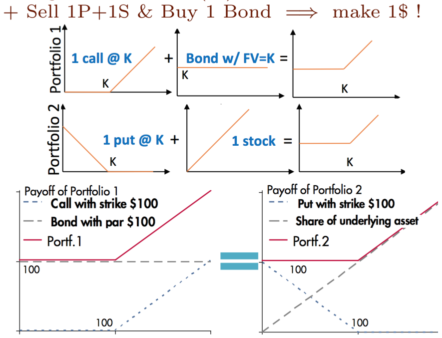
(Out Of The Money) Option NOT worth exercising: you will NOT profit. The strike price K of **Call** (**Put**) option is **above** (**below**) the market price of the underlying asset.

Theorem (Put-Call Parity)

$$C + PV(K) = P + S$$
$$C + \frac{K}{(1+r)^T} = P + S$$

(∵)No arbitrage ⇒ that 2 portfolios must have the same cost
Note: Use r=APR

Note: If arbitrage: $P + S - PV(K) = 5\$$ & $C = 4\$$
⇒ Buy 1C + Sell 1P+1S & Buy 1 Bond ⟹ make 1\$!

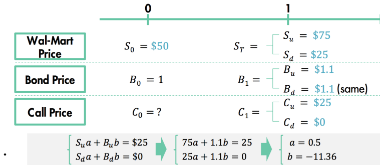


Binomial Asset Pricing

Idea: Know price of bond+stock; Want price of option.
⇒ set up a portfolio investing a in stocks and b in bonds that exactly replicates the payoffs from the option

Proposition No arbitrage ⇒ the price of option $\stackrel{!}{=}$ price of portfolio:

$$C_0 = aS_0 + bB_0 \quad \text{, where}$$
$$aS_u + bB_u = C_u$$
$$aS_d + bB_d = C_d \quad \text{, with}$$



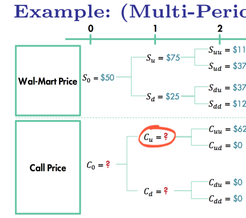
$B_0 = 1\$$, $B_u = B_d = 1\$ \cdot (1+r)^T$.
Solution:

$$a = \frac{C_u - C_d}{S_u - S_d}$$
$$b = \frac{C_u S_d - C_d S_u}{B_u S_d - B_d S_u}$$

The number of shares a needed to replicate a call option is called the **option's hedge ratio or delta.**

Proposition (American Optimal Strategy)
Optimal Strategy = max(option value if exercise, option value if don't)

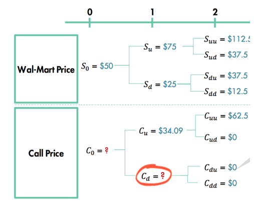
Example: (Multi-Period Example) $K = 50\$$, $T = 2y$



Replicating portfolio for $S_u = 75\$$:

$$112.5a + 1.1b = 62.5$$
$$37.5a + 1.1b = 0 \quad \text{, with}$$

So: $a = 0.833$, $b = -28.4$:
EU: $C_u = 0.833 \cdot 75 - 28.4 = 34.075$
US: C_u=max{34.075,75-50} = 34.075



Replicating portfolio for $S_d = 25\$$:

$$37.5a + 1.1b = 0$$
$$12.5a + 1.1b = 0 \quad \text{, with}$$

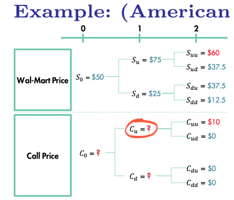
So: $a = 0$, $b = 0$:
EU: $C_d = 0$
US: $C_d = \max\{0, 25 - 50\} = 0$

Replicating portfolio for $S_0 = 50\$$:

$$75a + 1.1b = 34.09$$
$$25a + 1.1b = 0 \quad \text{, with}$$

So: $a = 0.682$, $b = -15.496$.
EU: C_0=0.682 · 50 − 15.496=18.60. US: C_0=max{18.60, 50-50}=0

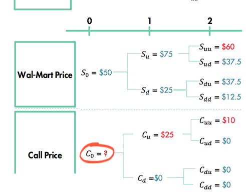
Example: (American Options) $K = 50\$$, $T = 2y$



Replicating portfolio for $S_u = 75\$$:

$$60a + 1.1b = 10$$
$$37.5a + 1.1b = 0 \quad \text{, with}$$

So: $a = 0.444$, $b = -15.15$:
EU: $C_u = 0.444 \cdot 60 - 15.15 = 18.18$
US: C_u=max{18.18,75-50} = 25

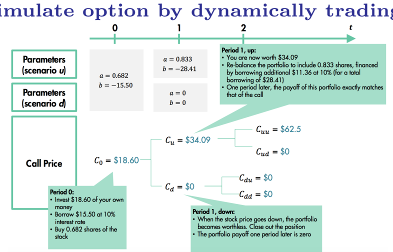


Replicating portfolio for $S_0 = 50\$$:

$$75a + 1.1b = 25$$
$$25a + 1.1b = 0 \quad \text{, with}$$

So: $a = 0.5$, $b = -11.36$:
EU: $C_0 = 9.92$ (using old EU price)
US: C_0=0.5 · 75-11.36=13.63 > 0 ✓

Example: (Simulate option by dynamically trading stock/bond)



Black-Scholes-Merton

__Limits of Binomial:__

- Trading takes place continuously!
- Price can take more than two possible values!

⇒ Let period-length get smaller: obtain Black-Scholes-Merton option pricing formula:

Theorem (Black-Scholes-Merton option pricing formula)

$$C(S,K,T,\sigma) = S \cdot N(x) - K \cdot (1+r)^{-T} \cdot N(x - \sigma\sqrt{T}) \quad \text{, with}$$

$$x = \frac{\log\left(\frac{S}{K(1+r)^{-T}}\right)}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T}$$

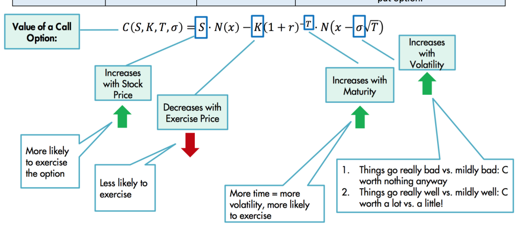
where:

- S = price of stock NOW
- r = annual riskless interest rate (units of a year).
- σ = volatility of annual returns on underlying asset.
- $N(.)$ = Gaussian CDF.

Idea:

- $C(S,K,T,\sigma)$: Call ≈ levered long position in the stock
- $N(x)$: number of shares (option delta)
- $S \cdot N(x)$: Amount invested in the stock
- $K \cdot (1+r)^{-T} \cdot N(x - \sigma\sqrt{T})$: Dollar amount borrowed

When … increases	What happens to value of call	What happens to value of put	Explanation
Strike price (K)	Decreases	Increases	When the strike price is close to the asset price the call is closer to be in the money. Opposite for the put.
Price of underlying asset (S)	Increases	Decreases	Opposite logic from the above. When the asset price increases, the call option (which is a long position) increases, as expected.
Volatility of underlying asset (σ)	Increases	Increases	Remember, option holders bet on volatility. High σ means high volatility, so more value for the option holder.
Maturity (T)	Increases	Increases	With more time available, there is a higher chance the option will eventually be in the money.
Interest rate (r)	Increases	Decreases	The call price is given by $C = S + P - \frac{K}{(1+r)^T}$. So when r goes up, C goes down. Vice versa for a put option.



Definition (Implied Volatility) Given the BSM formula, find σ. The VIX index is a good indicator of investor's fear over the years. if σ ⇑: good! get upside, not downside

Proposition BSM: Volatility increases the value of a Call Option:
−Things go really well vs. mildly well: Call worth a lot vs. a little!
−Things go really bad vs. mildly bad: Call worth nothing anyway

Equity & Debt as an Option

Corporate securities can be viewed as options:

Proposition

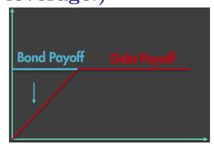
- Equity = call option on firm's assets (K = its bond's redemption value)
- Debt = portfolio combining the firm's assets + a short position in the call (with k = its bon's redemption value)

Proposition (Recipe for replicating payoffs with options)

1. Start with the risk-free bond or asset
2. Adjust the downside payoffs (put option) or upside payoffs (call option)
3. Adjust the payoffs up (buy option) or down (sell option)

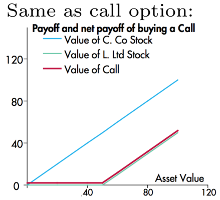
Example: (Debt in a firm with leverage:)

Start with bond
+ Adjust downside payoffs down
= Hold the bond and sell a put!



Example: (Stock in a firm with leverage:)

Same as call option:



Asset Value	Value of C. Co. Stock	Value of L. Ltd. Stock	Value of Call
⋮	⋮	⋮	⋮
$20	$20	$0	$0
40	40	0	0
50	50	0	0
60	60	10	10
80	80	30	30
100	100	50	50

__Corporate securities can be viewed as options:__

Equity: A **call option** on firm's assets (K = its bond's redemption value): Equity = max(Assets − Debt, 0)

Debt: A portfolio combining the firm's **assets** + a short position in the **call** (with K = its bond's redemption value):
Debt = min(Assets, Debt) = Assets − max(Assets − Debt, 0)

Warrant: A **call** option on the firm's stock

Convertible Bond: A portfolio combining **straight bonds** + a **call** on the firms stock (with K related to the conversion ratio)

Callable bond: A portfolio combining **straight bonds** and a **call** written on the bonds

Real Options

The ability to make decisions as you go ("option value") can greatly expand the value of a project or business! The option to:

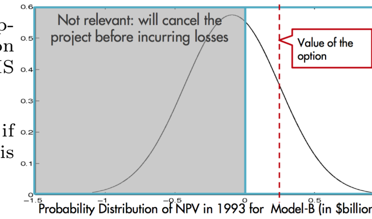
- Wait before investing
- Make follow-on investments
- Abandon a project
- Vary output/ production methods

Key elements in evaluating these options: New information arrives over time + Decisions can be made after receiving new information.

Example: (Follow-up projects are an example of real options)
1990: project A with $NPV = \$-46M$
1993: possible follow-up project B with $NPV = \$-92M$
In expectation, Model-B is a loser too. But there are scenarios in which Model-B really pays off! Should MC Inc. start Model-A?

Starting Model-B in 1993 is an option: so long as MC can abandon the business in 1993, only the RHS of the distribution is relevant.

NPV of the RHS is huge even if the chance of ending up there is less than 50%



Assume:

- Model-B decision has to be made in 1993
- Entry in 1993 with Model-A is prohibitively expensive
- MC has the option to stop in 1993 (possible loss limited)
- Investment needed for Model-B is $900M (twice that of A)
- PV(operating profits from Model-B)= $464M in 1990
- PV evolves with annual std of 39.5%; r_f = 6%.

Opportunity to invest in Model-B ≈ a 3y call option on asset worth $464M now (with K = $900M)!
BSM: Value of Call = $55M

Model	A	A+B
DCF	−$46M	−$99M
Option Value	$55M	
Total Value	$9M	−$99M

__Real options can change the investment decisions:__

Naive DCF analysis tends to under-estimate value of strategic options:

- Timing of projects is an option (American call)
- Follow-on projects are options (American call)
- Termination of projects are options (American put)
- Expansion or contraction of production are options (conversion options).

Recitation Examples

Risk and Return:

Example:

Expected return & β of security A + market portfolio.
CAPM holds: **find risk-free rate**

Security	$\mathbb{E}[\mathbf{r}]$	β
A	21%	1.6
Market	15%	1.0

CAPM $\Rightarrow r_A = r_f + \beta_A \cdot (r_m - r_f) \Rightarrow 21\% = r_f + 1.6 \cdot (15\% - r_f)$
$\Rightarrow \mathbf{r_f = 5\%}$

Example: $\beta_{ABC} = 0.4$, $r_f = 5\%$, market risk premium $= 6\%$.

- Find $\mathbb{E}[r_{ABC}]$: CAPM $\Rightarrow \mathbb{E}[r_{ABC}] = 5\% + 0.4\% \cdot 6\% = 7.4\%$
- $\sigma_m = 15\%$, $\rho(ABC, m) = 0.3$. Find σ_{ABC}.
  $\rho(ABC, m) = \frac{\text{Cov}(ABC,m)}{\sigma_{ABC}\sigma_m}$, $\text{Cov}(ABC, m) = \beta_{ABC}\sigma_m^2$
  $\Rightarrow \sigma_{ABC} = \beta_{ABC}\frac{\sigma_m}{\rho(ABC,m)} = 20\%$
- Expect $Price(ABC) = 150\$$ in 1yr. Find price today (p).
  $\mathbb{E}[r_{ABC}] = \frac{150\$ - p}{p} = 7.4\% \Rightarrow p = \frac{150\$}{1+7.4\%} = 139.66\$$

Example: Using the properties of CML & SML, are the following scenarios are (in)consistent with CAPM ?

Security	$\mathbb{E}[\mathbf{r}]$	β
A	25%	0.8
B	15%	1.2

Inconsistent: $\beta \uparrow \Rightarrow \uparrow \mathbb{E}[r]$
Not inconsistent if historical data

Security	$\mathbb{E}[\mathbf{r}]$	$\sigma(\mathbf{r})$
A	25%	30%
M	15%	30%

Inconsistent: A lies above CML
$\Rightarrow$ inefficient market portfolio
$SR_m = \frac{\mathbb{E}[r_m] - r_f}{\sigma_m} < \frac{\mathbb{E}[r_A] - r_f}{\sigma_A} = SR_A$

Security	$\mathbb{E}[\mathbf{r}]$	$\sigma(\mathbf{r})$
A	25%	55%
M	15%	30%
F	5%	0%

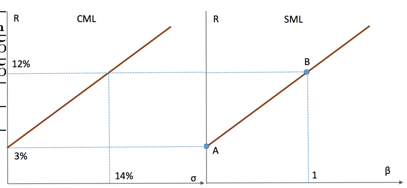
Inconsistent: A lies above CML
$\Rightarrow$ inefficient market portfolio
$SR_m = 0.36 < 0.33 = SR_A$

Security	$\mathbb{E}[\mathbf{r}]$	β
A	20%	1.5
M	15%	1.0
F	5%	0

Consistent: A lies on SML
$r_f + \beta_A \cdot (r_m - r_f) = r_A$ ✓

Example:

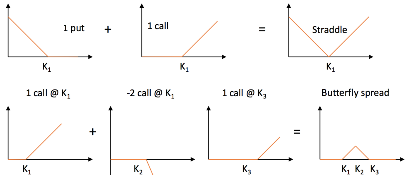
Stock	$\mathbf{r_i}$	σ_i	ρ_{im}
1	6.825 %	17%	0.35
2	22.125 %	35%	0.85
M	12%	14%	1
F	3%	0%	0



1) Find $\mathbb{E}[r_1]$ & $\mathbb{E}[r_2]$:
$\beta_{1m} = \rho_{1m}\frac{\sigma_1}{\sigma_m} = 0.425$
$\beta_{2m} = \rho_{2m}\frac{\sigma_2}{\sigma_m} = 2.125$
−CAPM Eq: $r_i = r_f + \beta_{im}(r_m - r_f) \Rightarrow r_1 = 6.825\%$ & $r_1 = 22.125\%$
−Geometry: SML line $r_i = 9\beta_{im} + 3 \Rightarrow r_1 = 6.825\%$ & $r_1 = 22.125\%$
Intuition:
$\beta_{1m} = 0.425 \Rightarrow$ stock risk prem. should be half of market risk prem.
$\beta_{2m} = 2.125 \Rightarrow$ stock risk prem. should be double of market risk prem.
2) Given $\rho_{12} = 0.85$, $w_1 = 15\%$ & $w_2 = 85\%$:
$\mathbb{E}[r_p] = w_1\mathbb{E}[r_1] + w_2\mathbb{E}[r_2] = 19.84\%$
$\sigma(r_p) = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{12}\sigma_1\sigma_2} = 31.95\%$
3) Construct portfolio with same return as r_p & lower risk
$0.1984 = \mathbb{E}[r_p] = w_1\mathbb{E}[r_m] + w_2\mathbb{E}[r_f] = 0.12w_1 + 0.03(1 - w_1)$
$\Rightarrow w_1 = 187\%$ $w_2 = -87\%$ (borrow at rf rate to lever up)
$\sigma_{new} = w_1\sigma_m = 26.18\%$ OR use geometry (CML):
$r = a\sigma + b \Rightarrow r_i = 0.643\sigma_i + 3 \overset{!}{=} 19.84\% \Rightarrow \sigma_{new} = 26.18\%$

Options:

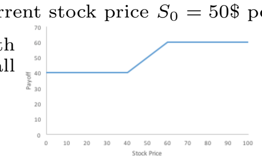
Example: (Payoff Profiles)



Area of curve: large+extends far from strike price on both directions $\Longrightarrow$ bet on volatility.
O'wise: expect price to move within a short range around strike price.

Let $T = 6-$month maturity & current stock price $S_0 = 50\$$ per share.

(a) Buy 1 share + buy 1 put with exercise price $K = 40\$$ + sell 1 call with exercise price $K = 60\$$.



(b) Buy 1 put + 1 call with exercise price $K = 50\$$ + sell 1 put with exercise price $K = 40\$$ + sell 1 call with exercise price $K = 60\$$.

Example: (Real Options)

You financed a new movie for $PV = \$75M$. You can do a sequel which would cost $\$100M$ to produce and could be made any time in the next five years.

1. If we think of the possibility of making a sequel as an option, is it a call or a put? European or American?
   Solution: *American call*
2. Strike price & time to expiration/maturity?
   Solution: $K = \$100M, T = 5y$

   Sequel will produce 15% less CFs than 1st movie produces (e.g. if 1st produces $\$100M$, 2nd will produce $\$85M$).
3. Find minimum free CF (excluding the initial investment) generated by 1st movie for which you'd exercise the option and make the sequel?
   Solution: *You would exercise the option when 1st movie is expected to make more money than it costs to produce:*
   $0.85 \cdot CF \geq \$100M \implies CF \geq 100M/0.85 = \$117.65M$