Introduction

Principles: Value of an **Asset** = Value of its **Cash Flows**.

More $ > Less $ – Investors Prefer More to Less

Now $ > Tomm $ – $ paid in future is worth < same amount today

Safe $ > Risky $ – Investors are risk averse

No Arbitrage – Financial markets are competitive

Def. (Arbitrage) Transaction where profit is made with no extra risk by simultaneously buying and selling an asset that is not properly priced in 2 markets./ Buying smth at a price, sell it at other price, without risk, when cost of doing the transaction<difference in price.

Net Present Value

CFs are discounted for two reasons: 1$ today is worth more than 1$ tomorrow + A safe 1$ is worth more than a risky 1$.

Present Value (PV) - How much is a cash flow at time t (CF_t) worth today ($PV(CF_t)$) using discount rate r ?

$$PV(CF_t) = \frac{CF_t}{(1+r)^t}$$

Net Present Value (NPV) - Value entire stream of CFs ?

$$NPV(CF_0, CF_1, \cdots, CF_T) = CF_0 + \frac{CF_1}{(1+r)} + \cdots + \frac{CF_T}{(1+r)^T}$$

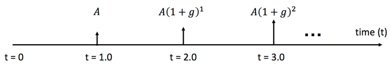
Thm. A project is good $\iff NPV > 0$

Assumptions CFs known (amount & time) + r known & cst.

Ppties X_t, Y_t cash flows, $a \in \mathbb{R}$:
$NPV(a \cdot X_0, a \cdot X_1, ..., a \cdot X_T) = a \cdot NPV(X_0, X_1, ..., X_T)$
$NPV(X_0 + Y_0, ..., X_T + Y_T) = NPV(X_0, ..., X_T) + NPV(Y_0, ..., Y_T)$
$NPV(X_0, ..., X_T) = NPV(X_0, ..., X_\tau) + NPV(X_{\tau+1}, ..., X_T)$

Discount Rate r Determined by rates of return prevailing in capital markets.

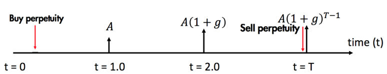
Future Value (FV) - How much is a CF today (PV_{CF}) worth at time t (FV_t) at rate of return r ? $FV_t = PV_{CF} \cdot (1+r)^t$



Def. (Perpetuity) Infinite stream of fixed CFs

Prop. $CF_t = A$ for all $t \geq 1$: $PV(Perpetuity) = \frac{A}{r}$

$PV(Perpetuity) = \frac{A}{r-g}$ if constant growth g



Def. (Annuity) Finite stream of fixed CFs

Prop. $CF_t = A$ for all $1 \leq t \leq T$:

$PV(Annuity) = \frac{A}{r}\left(1 - \frac{1}{(1+r)^T}\right)$ buy perp at $t=0$, sell at $t=T$

$PV(Annuity) = \frac{A}{r-g}\left(1 - \frac{(1+g)^T}{(1+r)^T}\right)$ if cst growth $g < r$

$PV(Annuity) = T \cdot \frac{A}{1+r}$ if cst growth $g = r$

More frequent compounding → higher return.

Discount Rate: r s.t. indiff betw 1$ now & $(1+r)$$ after 1 period.

Interest Rate: r that makes NPV equation hold (given CF)

Link : I.R.=representative of average "market" discount rates. Your personal D.R. may be affected by I.R. through opport. cost of invest.

Def. (Annual Percentage Rate, APR) :

APR = Interest rate per period (e.g., month) × # of periods in a year.

Prop. Invest 1$ at **APR** $= r$ **compounded** m **times per year**:

- Investment at end of year worth $= \left(1 + \frac{r}{m}\right)^m \times 1\$$.

- Effective interest rate $(EAR/APY) = \left(1 + \frac{r}{m}\right)^m - 1$.

- $APR = r = m\left[(1+EAR)^{\frac{1}{m}} - 1\right]$ (m = # of periods in a year)

Note: If interest is paid evenly throughout the year: interest often quoted as a **continuously compounded rate**: $(1+EAR)^1 = e^{APR}$

"Nominal" rates of return are the prevailing/quoted market rates. Nominal CFs are expressed in **dollars at each date**.

"Real" rates are nominal rates adjusted for inflation. Real CFs are expressed in **constant purchasing power**.

Ex: (Nominal vs. real cash flows) Infl= 4% per yr. Expect to receive 1.04$ in 1 year. This CF is really worth next year:
$(\text{Real CF})_t = \frac{(\text{Nominal CF})_t}{(1+i)^t} = \frac{1.04\$}{1+0.04} = 1\$$

Def. (Inflation Risk) CFs measured in nom. terms ($)→ RCFs exposed to infl risk (risky even if delivery is certain). Exposure to infl risk depends on financial position: Lenders lose/Borrowers gain.

Ex: (Nominal vs. real rates of return) :
−Nominal rates of return are the prevailing market interest rates
−Real rates of return are the inflation adjusted rates.
1.00$ invested at a 6% interest rate grows to 1.06$ next year.
However, if inflation is 4% per year, then the real rate of return is:
$r_{real} = \frac{1+r_{nominal}}{1+i} - 1 = \frac{1+0.06}{1+0.04} - 1 = 1.9\%$
$g_{real} = \frac{1+g_{nominal}}{1+i} - 1$ **Note:** $r_{real} \approx r_{nominal} - i$

Capital Budgeting

Thm. (NPV Rule) Project with CFs $\{CF_0, CF_1, \cdots, CF_T\}$:
Current Market Value: $NPV = CF_0 + \frac{CF_1}{1+r} + ... + \frac{CF_T}{(1+r)^T} > 0$.

1. Use cash flows attributable to the project: Use incremental CFs / Forget sunk costs / Include investment in WC and in CapExp / Include opportunity costs of using existing facilities / Be consistent in treatment of inflation
2. Use (after-tax) CFs, not accounting earnings

$r = 14\%, \tau = 35\%$	0	1	2	3	4	5
CapEx	500					
Revenue		500	500	500	500	500
Expenses		300	300	300	300	300
EBITDA = Rev-Exp		200	200	200	200	200
Depreciation		100	100	100	100	100
Salvage Value						0
Working Capital	50	50	50	50	50	0
ΔWC	50	0	0	0	0	-50
CF	-550	165	165	165	165	215
PV	-550	143.5	124.8	108.5	94.3	106.9
Total NPV	+28					

WC only during project: zero at the end (unless proj goes on forever)

Def. (Capital Expenditures, CapEx) Funds used by a company to acquire or upgrade physical assets such as property, industrial buildings or equipment. It can include everything from repairing a roof to building, to purchasing a piece of equipment, or building a brand new factory.

Accounting: **Depreciate** CapEx linearly over multiple periods. Depreciate **more**: smaller loss/looks better; **less**: larger CFs.
Warning: Depreciation matters for CFs through taxes!

- $CF = (1-\tau) \times EBITDA - CapEx + \tau \times Depr. - \Delta WC + [Salv.Val - \tau \times (Salv. - BookVal)]$
- $CF = (1-\tau) \times EBIT - CapEx + Depr. - \Delta WC + [Salv.Val - \tau \times (Salv. - BookVal)]$

CF = Project cash Inflows − Outflows.
DO NOT depreciate (not a CF), but include CapExp

Non CapExp = COGS + OpExp

- **Cost of Goods Sold:** COGS = direct costs attributable to the production of the goods sold by a company. This amount includes the cost of the materials used in creating the good along with the direct labor costs used to produce the good.
- **Operating Expenses:** An expense incurred in carrying out an organization's day-to-day activities, but not directly associated with production. Operating expenses include such things as payroll, sales commissions, employee benefits and pension contributions, transportation and travel, amortization and depreciation, rent, repairs, and taxes.

Oper. Profit/Income = Oper. Rev.−Non CapExp w/o Depr−Depr

- **Taxes** Income taxes = $\tau \times$ Op.Prof.
- **Operating Revenues:** = $\tau \times$ Op.Prof.

Def. (EBITDA) Earnings Before Interests, Taxes, Depreciation, and Amortization (=depr for intangible assets: patents...)
Measures how much cash is coming in now.
EBIT: Earnings Before Interests & Taxes

Def. (Working Capital WC) = Assets − Liab.
WCap = Inventory + Acc.Receivable − Acc.Payable
Measures company's liquidity, efficiency, and overall health:
WC = money available to a company for day-to-day operations.

- **Inventory**: Raw materials, work-in-progress products + finished goods considered ready for sale (or soon).
  COGS ignores cost of items produced but not sold.
  Inventory ↑: COGS understates cash outflows;
  Inventory ↓: COGS overstates cash outflows.
- **Accounts Receivable:** Money owed to the firm for sales by its clients/customers. Accounting sales may reflect sales that have not been paid for. Accounting sales understate cash inflows if the company is receiving payment for sales in past periods.
- **Accounts Payable:** reverse of Accounts Receivable.
- **Change in WC,** ΔWC Measures delays between recorded sale and cash received.
  $\Delta WC > 0$ (< 0) : company (UN)able to pay off its short-term liabilities almost immediately. Equal ↑ in CF.
  $\Delta WC < 0$ suggests a company is becoming over-leveraged, struggling to maintain or grow sales, paying bills too quickly, or collecting receivables too slowly (opposite if $\Delta WC > 0$). Equal ↓ in CF.

Possible sources of positive NPV: Short-run competitive advantage (right place at the right time) + Long-run competitive advantage (patent, technology, economies of scale, etc.).

Def. (IRR) = discount rate that makes the projects NPV equal to 0:
$CF_0 + \frac{CF_1}{(1+IRR)} + \frac{CF_2}{(1+IRR)^2} + \ldots + \frac{CF_k}{(1+IRR)^k} \overset{!}{=} 0$. $IRR > IRR^* \Rightarrow$ ✓

Def. (Payback Pd) =min k s.t $CF_1 + . + CF_k \geq -CF_0$;$k < k^* \Rightarrow$ ✓

Def. (Discounted Payback Meth) =min k s.t
$\frac{CF_1}{(1+r)} + . + \frac{CF_k}{(1+r)^k} \geq -CF_0$; $k < k^* \Rightarrow$ ✓

Def. (Profitability Index) $PI = \frac{PV(CF_{1,.})}{-CF_0}$; $PI \geq 1 \Rightarrow$ ✓

Bonds

Prop. (Bond) Price(bond)=PV(remaining CFs) (coupons+principal)

Def. (Spot (interest) Rates) r_t = current (**annualized**) rate for payments in t periods from now.

Def. (STRIP/Discount/0-Coupon Bond) Pays $FaceVal = 1\$$ only at maturity t: Price $B_t = \frac{FV}{(1+r_t)^t}$; $r_t = \left(\frac{FV}{B_t}\right)^{1/t} - 1$ is spot rate

Def. (Coupon Bd,YTM=y) $Price = C_1B_1 + ... + (C_t + FV)B_t = \frac{C_1}{(1+r_1)^1} + ... + \frac{C_{t-1}}{(1+r_{t-1})^{t-1}} + \frac{FV+C_t}{(1+r_t)^t} = \frac{C_1}{(1+y)^1} + ... + \frac{C_{t-1}}{(1+y)^{t-1}} + \frac{P+C_t}{(1+y)^t}$

Prop. ($\mathbb{E}$xp Hypothesis) $r_{t,m}$ = rate on maturity m at date t. Then: $(1 + r_{t,m})^m = (1 + r_{t,1})(1 + \mathbb{E}_t[r_{t+1,1}]) \ldots (1 + \mathbb{E}_t[r_{t+m-1,1}])$

Prop. (LiqPrefHyp) $(1 + r_{t,2})^2 = (1 + r_{t,1})(1 + \mathbb{E}_t[r_{t+1,1}]) + \text{LqPrm}$

Int.Rt.Rsk: The more you compound, the more Δr affects ΔB, so: Longer Bonds $\Rightarrow$ Larger Risks but safer hold to maturity than roll over

Duration measures interest rate risk exposure: Assume: $r_t = y$

Def. (Macaulay Duration) Bond Price $B = \sum_{t=1}^{T} \frac{CF_t}{(1+y)^t} \Rightarrow$ $D = \sum_{t=1}^{T} \frac{PV(CF_t)}{B} \times t = \frac{1}{B} \sum_{t=1}^{T} \frac{CF_t}{(1+y)^t} \times t$,

Def. (Modified Duration) $MD = -\frac{1}{B}\frac{\Delta B}{\Delta y} = \frac{D}{1+y}$ $\Rightarrow \Delta B\% = \frac{\Delta B}{B} \approx -MD \cdot \Delta y$ yield $\Uparrow$ price $\Downarrow$

Prop.
- D = weighted avg time it will take to get your payments
- MD = measures how sensitive the price of a bond ΔB is to market interest rates Δr. ex: $MD = 8$, so $r \Uparrow 1\%$ (e.g., $5-6\%$) $\Rightarrow$ $B \Downarrow 8\%$
- $D_P = \sum_x \frac{PV_x}{PV_P} \times D_x$ for Portfolio P of bonds x
- D(0-Coupon Bd) = maturity m, D(perp) = $\frac{1+y}{y}$ perp dbt of yield y
- $D \leq$ maturity always, and $D \leq$ maturity of 0-coup bd of same dur^o
- Coupon rate $\Uparrow$ and all else equal $\Rightarrow$ $D \Downarrow$
- YTM $\Uparrow$ and all else equal $\Rightarrow$ $D \Downarrow$ ($\because$) discount future more
- Immediately after a coupon payment, D_{bond} decreases ? NO!
- Term structure of interest rates $\Uparrow \Rightarrow$ investors expect higher short term interest rates in future ? No/Uncertain

Def. (Portfolio Immunization/Duration Matching) Make $D_A = D_L \Rightarrow$ interest rate changes makes the $\Delta P_A = \Delta P_L$ $MD_{assets} \times PV_{assets} = MD_{liab} \times PV_{liab}$ $(\because)P$ = Price: $\Delta P \approx -\frac{D_{assets}}{1+y} P_{assets}\Delta y + \frac{D_{liab}}{1+y} P_{liab}\Delta y \stackrel{!}{=} 0$

Price risk: int rate $\Downarrow \Rightarrow$ PV(bonds) $\Uparrow \Rightarrow$ PV(liab) $\Uparrow$ more!

Reinvestment risk: At new interest rate, assets cannot be reinvested to make future payments

Ex: (Immunize Your Portfolio) $x = \#$ of 1yr bonds with price P_1, $y = \#$ of 30yr bonds with price P_{30}, PV(Portfolio) $= P$. $P = xP_1 + yP_{30}$ and $MD_P P \stackrel{!}{=} 1Yr \times \frac{x}{1+r} + 30Yr \times \frac{y}{(1+r)^{30}}$

Ex: (Gamble:$D_A < D_L$ = bet that rates will rise) buy $P_A = 100\$$ of 1Y bd, and sell $P_L = 100\$$ of 10Y bd. If rates rise by $\Delta y = 1\%$: $\Delta B_1\% = -MD\Delta y \approx -1 \times 1 = 1\%$ (assume $MD \approx D$) and $\Delta B_{10}\% = -MD\Delta y \approx -10 \times 1 = 10\%$. Then $\Delta B_1 = -1\$$ and $\Delta B_{10} = -10\$ \Rightarrow 9\$$ profit.

Def. (Inflation Risk) Most bonds give nominal payoffs: inflation risk $\Rightarrow$ risky real payoffs (even if nominal payoffs are safe).

Def. (Default/Credit Risk:) Risk that dbt issuer fails to make the promised payments. Bd ratings by rating agency: Moody, S&P, Finch

Capital Structure

Def. (Assets/Value) Assets = Equity + Debt

Def. (Leverage Ratio) $LR = \frac{\text{Assets}}{\text{Equity}} = \frac{E+D}{E}$

Note: $E = V - D$ so $\Delta E = \Delta V$ and $\frac{\Delta E}{E} = \frac{V}{E} \times \frac{\Delta V}{V} = LR \times \frac{\Delta V}{V}$

Def. (Capital Structure:) Optimal combination of D&E ? MM: NO!

Thm. (MM) Conditions: Complete/efficient market, no taxes, same r for all assets, no transaction/distress/bankruptcy costs, firm behavior not affected by capital structure. $\Rightarrow$ THEN: The market value of any firm is independent of its capital structure: any combination of securities is as good as another. If we assume no tax shield from debt & $r_E = r_D$ (usually $r_E > r_D$ as $t_D < t_E$) & CFs indep. of financing:

$$V = \sum_{t=1}^{\infty} \frac{CF_t}{(1+r)^t} = \sum_{t=1}^{\infty} \frac{CF_t - rD}{(1+r)^t} + \sum_{t=1}^{\infty} \frac{rD}{(1+r)^t} = E + D$$

Def. (Rate of Return) = Payment/Price

r_V, r_D, r_E: Return to buying asset, collecting a payment, selling asset.

Def. (Leverage Ratio) $\lambda = \frac{\text{Debt}}{\text{Assets}} = \frac{D}{E+D}$

Assume a firm wants to invest using a constant leverage ratio: Debt D, Value of Assets V; **Constant leverage** $\Rightarrow$ $D = \lambda V$

Interest Rate on Debt/Cost of Debt: $r_D = \frac{r_D D}{D}$

Cost of Equity: $r_E = \frac{(1-\tau)(CF - r_D D)}{E}$

Rate of Return of the Firm: $r_V = \lambda r_D + (1-\lambda)r_E$

$(\because) r_V = \frac{(1-\tau)(CF - r_D D) + r_D D}{V} = \frac{r_D D}{V} + \frac{(1-\tau)(CF - r_D D)}{E}$

Note: Usually $r_E > r_D$ as r_E is riskier than r_D (D paid before E). Debt changes/borrow more $\Rightarrow r_D$ & r_E move.

Note: Shareholders are indifferent to increase in leverage ratio: $\lambda \uparrow \Rightarrow$ $\mathbb{E}$[returns] $\uparrow \Rightarrow$ Risk $\uparrow$

Prop. If CFs are constant (i.e., $CF_t = CF$ $\forall t$), the **Total Firm Value** is:

$$V = \frac{(1-\tau)CF}{(1-\tau)\lambda r_D + (1-\lambda)r_E} \stackrel{!}{=} \frac{(1-\tau)CF}{r_{WACC}}$$

Proof: $V = \sum_{t=1}^{\infty} \frac{(1-\tau)CF + \tau r_D D}{(1+r_V)^t} = \frac{(1-\tau)CF + \tau r_D D}{r_V}$. Use $D = \lambda V$ and $r_V = \lambda r_D + (1-\lambda)r_E$. □

Def. (Weighted Average Cost of Capital, WACC) Discount rate used to calculate project NPV for firm. How much should the firm make on 1\$ to be able to repay debts r_D and meet the required rate of return r_E for investors ? Lower WACC is better!

$$r_{WACC} = (1-\tau)\lambda r_D + (1-\lambda)r_E = (1-\tau)r_D \frac{D}{D+E} + r_E \frac{E}{D+E}$$

Prop. (General WACC Formula)

$$V = \sum_{t=1}^{\infty} \frac{(1-\tau)CF_t}{(1+r_{WACC})^t}, \text{ and } E = V - D$$

Note: Value$\uparrow$ $\Rightarrow$WACC$\downarrow$

Rmks:
- All NPV accrues to equity holders.
- Leverage ratios should be the project's operational target
- Discount rates are project-specific. Each project like stand-alone firm.
- Cost of debt could be estimated with current market rate charged to comparable firms with similar credit risk
  - If the project has different layers of debt, an average cost of debt should be estimated
- If the project capital structure is expected to remain stable, then WACC will be stable. Otherwise, WACC should change.
  - In practice, firms tend to use a stable WACC regardless.
- Optimal capital structure: choose λ to maximize NPV.
  - Why not all debt ? interest rate r_D would increase.

Stocks

Def. (Common Stocks) Represent equity/ownership positions in a corporation + give right to payments made in various forms: Cash dividends, Stock dividends, Share repurchases. $\Rightarrow$ payments are not certain in timing + magnitude: depend on firm's performance & policy. Legal rights: Residual claim, Limited liability, Voting rights.
ex: Preferred Stocks: Generally no voting rights but prior claim on earnings and assets.

Def. (Trading in secondary market:) Trading costs, Buy on margin, Long, Short Position/Short Selling

Def. (Discounted Dividend Model) Stock price: $P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+r_t)^t}$ where P_t = Stock price at t ex-dividend, D_t = expected cash dividend at t, $r_t = CF_t$ risk-adjusted discount rate.

Def. (Gordon Growth) Constant Growth Discounted Divid Model: (r constant, D_t grow in perpetuity at constant rate $g < r$) $P_0 = \sum_{t=1}^{\infty} \frac{(1+g)^t D_1}{(1+r)^t} = \frac{D_1}{r-g} = \frac{D_0 \times (1+g)}{r-g}$

Def. (Dividend Yield) $= \frac{D_0}{P_0}$ or $\frac{D_1}{P_0}$

Def. (Implied Cost of Capital) $r = \frac{D_1}{P_0} + g = \frac{D_0}{P_0}(1+g) + g$ where g =growth rate of dividends in long run $\Rightarrow$CoC=Div.Yld+Div.Grwth

Earnings E = Prof. $-$ Depr. = Dividends + New investments - Depr.

Earnings (per share): EPS = total profit net of depr. and taxes

Payout ratio: p = Dividends/Earnings $= \frac{DPS}{EPS}$

Retained earnings: RE =Earnings$-$Dividends $= \Delta BV$

Plowback ratio: $b = 1 - p = RE$/Earnings

Book value: BV = Cumulative RE $= BV_{start} + EPS \times b$ $= BV_{start} + E - D$, $BVS_1 = BVS_0 \times (1+g)$

Return on book equity: ROE = Total Earnings/BV

Time:	1	2	3
$BV_{start} = \frac{EPS}{ROE}$	20	23.6	ignore
$BV_{end} = BV_{start} + EPS \times b$	23.6	ignore	ignore
$ROE = \frac{EPS}{BV_{start}}$	**0.2**	**0.15**	**0.15**
$p = 1 - b = \frac{DPS}{EPS}$	0.1	**0.5**	**0.5**
$b = 1 - p = \frac{RES_1}{EPS_1}$	**0.9**	0.5	0.5
$EPS = BV_{start} \times ROE$	**4**	3.54	ignore
$DPS = EPS \times p = DPS_{-1}(1+g)$	0.4	1.77	1.90
$g = ROE \times b = \frac{\Delta BV}{BV}$	not cst	**0.075**	**0.075**
Time:	0	1	2
DPS	–	0.4	1.77
$Perp = \frac{D}{r-g}$	–	1.77/(1+0.075)	
$PV = PV(D_1) + PV(Perp)$	64.73		

Prop. $Earnings = BV_{-1} \times ROE$ and $RE = b \times BV_{-1} \times ROE$ $Dividends = Earnings - RE = (1-b) \times BV_{-1} \times ROE =$ $p \times BV_{-1} \times ROE$ **Note:** b & ROE cst $\Rightarrow$ Div. & BV grow at same g

Note: If $ROE = r$: growth plan irrelevant, price will be the same. If $ROE > r$: $g \Uparrow$ ‖ $ROE < r$: $g \to 0$‖ ROE & g cst: $RES_1 = g \times BV_0$, $D_1 = (ROE - g)BVS_0$, and $P_0 = \frac{(ROE-g)BV_0}{r-g}$

Def. (Growth Opportunities) Investment opportunities that earn expected returns higher than the cost of capital ($ROE > r$) **Growth Stocks, GS:** Stocks of companies that have access to growth opportunities.
NOT necessarily GS: a stock with growing EPS, Dividends or Assets.
Maybe GS: stock with EPS(or DPS)growing slower than cost of capital

Def. (PVGO) Earnings under a no-growth policy: price $P_0 = \frac{EPS_1}{r}$
Grwth: $P_0 = \frac{EPS_1}{r} + PVGO \stackrel{!}{=} \frac{D_1}{r-g} = \frac{p \times ROE \times BV_0}{r-g} = \frac{(ROE-g)BV_0}{r-g}$

Prop. $PVGO < 0 \Leftrightarrow ROE < r$:firm should distribute \$ as div.

Earning Yield: $\frac{EPS}{P} = \frac{EPS_1}{P_0}$; **PE Ratio** $\frac{P}{E} = \frac{P_0}{EPS_1}$ (higher for GS)

Note: Which have higher returns on average, growth stocks or non-growth ("value") stocks?

Pai Mei: Legend. master of Bak Mei+Eagle's Claw (kung fu styles). +1000 years old & poisoned in 2003. Eats fish heads.

By Zied Ben Chaouch

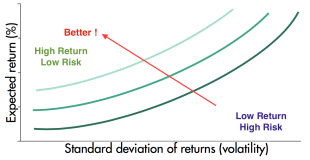
Diversification

$\mathbf{P_0}$&$\widetilde{\mathbf{P_1}}$&$\widetilde{\mathbf{D_1}}$: Price/Div at the beginning/end of the period.
Note: Divs less risky than price (D_1 depends mainly on this time period, while P_1 aggregates info&predictions about future of the firm)

$\widetilde{\mathbf{r_1}} = \frac{\widetilde{D_1}+\widetilde{P_1}}{P_0} - 1 =$ Return. $\mathbf{E}[\widetilde{\mathbf{r_1}}] =$ Expected Return.
$\widetilde{\mathbf{r_1}} - \mathbf{r_F} =$ Excess Return. $\mathbf{E}[\widetilde{\mathbf{r_1}}] - \mathbf{r_F} =$ Risk Premium.
Mean: $\overline{r_1} = E[\tilde{r_1}] = \sum_i p_i r_i$. Estimator: $\widehat{r} = \frac{1}{T}\sum_{t=1}^{T} r_t$.
Var: $\sigma^2 = \text{Var}(\tilde{r_1}) = E[(\tilde{r_1} - \overline{r})^2] = \sum_i p_i(r_i - \overline{r})^2$.
Estimator: $\widehat{\sigma^2} = \frac{1}{T-1}\sum_{t=1}^{T}(r_t - \widehat{r})^2$. **Std:** $\sigma = \sqrt{\sigma^2}$.
Cov: $\sigma_{ij} = \text{Cov}(\tilde{r_i}, \tilde{r_j}) = E[(\tilde{r_i}-\overline{r_i})(\tilde{r_j}-\overline{r_j})] = \sum_i p_i(r_i^a - \overline{r^a})(r_i^b - \overline{r^b})$
$\sigma_{ij} > 0 \Rightarrow \uparrow\uparrow$ / $\downarrow\downarrow$ and $\sigma_{ij} < 0 \Rightarrow \uparrow\downarrow$ / $\downarrow\uparrow$.
Correlation: $\rho_{ij} = \text{Corr}(\tilde{r_i}, \tilde{r_j}) = \frac{\text{Cov}(\tilde{r_i}, \tilde{r_j})}{\sigma_i \sigma_j}$ ranges from -1 to 1.
Beta: $\beta_{ij} = \frac{\text{Cov}(\tilde{r_i}, \tilde{r_j})}{\sigma_j^2}$. $\tilde{r_j}$ goes up by $1 \Rightarrow \tilde{r_i}$ goes up by β_{ij} on avg

Ttl Val of a Port: $V = \sum_i N_i P_i$ **Port Weight:** $w_i = \frac{N_i P_i}{N_1 P_1 + \ldots + N_n P_n}$
with $w_1 + \ldots + w_n = 1$. $w_i > 0 (< 0)$: long (short) position
Idea: Like: Higher expected returns/ Don't like: Higher risks

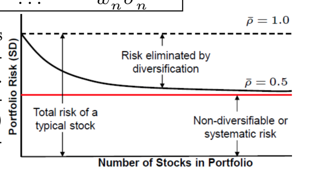
Indifference curve: set of return & volatility $(E[\tilde{r}], \sigma)$ combos that give an investor the same $\mathbb{E}$[utility].
Mean-Variance Utility: $U(\tilde{r}) = \mathbb{E}[\tilde{r}] - \frac{1}{2} \cdot A \cdot \text{Var}[\tilde{r}]$
$A > 0$:risk aversion. $A = 0$:risk neutral/care about $\mathbb{E}$[ret] not risk



$\Rightarrow$ Investor indiff betw: get return U for sure & gamble on return $\tilde{r}$.
Optimal portfolio reconciles what is desirable (indifference/ utility curves) with what is feasible (efficient frontier)
$r_p = w_1 r_1 + \ldots + w_n r_n$ $\mathbb{E}[\tilde{r_p}] = w_1\mathbb{E}[\tilde{r_1}] + \ldots + w_n\mathbb{E}[\tilde{r_n}]$
$\text{Var}[\tilde{r_p}] = \mathbb{E}\left[(\tilde{r_p} - \overline{r_p})^2\right] = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \text{Cov}[\tilde{r_i}, \tilde{r_j}]$
To compute covariance, add up all of entries in:

	$w_1 r_1$	$w_2 r_2$	...	$w_n r_n$
$w_1 r_1$	$w_1^2 \sigma_1^2$	$w_1 w_2 \sigma_{12}$	...	$w_1 w_n \sigma_{1n}$
$w_2 r_2$	$w_2 w_1 \sigma_{21}$	$w_2^2 \sigma_2^2$	...	$w_2 w_n \sigma_{2n}$
...	...	...	...	...
$w_n r_n$	$w_n w_1 \sigma_{n1}$	$w_n w_2 \sigma_{n2}$	...	$w_n^2 \sigma_n^2$

Diversification reduces risks: as long as $\rho \neq 1$, can get higher returns with lower risks ($\sigma_p < \sigma_i \, \forall i$).
Non-Diversifiable risk (or market/ systematic/ common risk) comes from Business cycle, Inflation, Volatility, Credit, Liquidity...



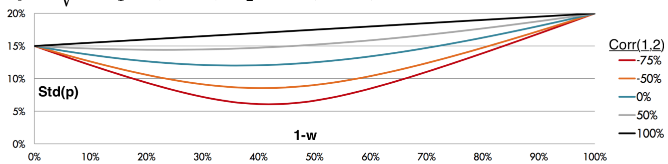
Prop. (Diversify to ∞) Portfolio with n assets **equally weighted**:
$\sigma_p^2 = \frac{1}{n}\left(\frac{1}{n}\sum_{i=1}^{n}\sigma_i^2\right) + \frac{n^2-n}{n^2}\left(\frac{1}{n^2-n}\sum_{i=1}^{n}\sum_{j\neq i}^{n}\sigma_{ij}\right)$
$= \frac{1}{n}$ (avg variance) $+ \frac{n^2-n}{n^2}$ (avg covariance) $\longrightarrow 0 +$ avg covariance

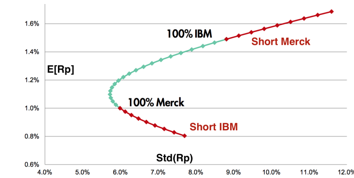
Def. (Lowest Risk (1)/Highest Return (2) Approach)
Minimize the total risk/volatility (1) or **Maximize** the expected return (2) of the portfolio **subject to**: Having a balanced portfolio + Achieving the desired level of return (1) or risk (2)

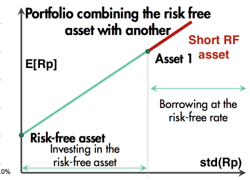
$$\min_{\{w_1,\ldots,w_n\}} \quad \sigma_p^2 = \sum_{i,j=1}^{n} w_i w_j \sigma_{ij}^2$$
$$\text{Subject To} \quad \sum_{i=1}^{n} w_i = 1, \quad \sum_{i=1}^{n} w_i \mathbb{E}[r_i] = \mathbb{E}[r_p]$$

$$\min_{\{w_1,\ldots,w_n\}} \quad \mathbb{E}[r_p] = \sum_{i=1}^{n} w_i \mathbb{E}[r_i]$$
$$\text{Subject To} \quad \sum_{i=1}^{n} w_i = 1, \quad \sum_{i,j=1}^{n} w_i w_j \sigma_{ij}^2 = \sigma_p^2$$

Ex: Two assets: $\sigma_1 = 15\%$ (weight w), $\sigma_2 = 20\%$ ($w_2 = 1 - w$).
$\sigma_p = \sqrt{w^2\sigma_1^2 + (1-w)^2\sigma_2^2 + 2w(1-w)\sigma_{12}}$



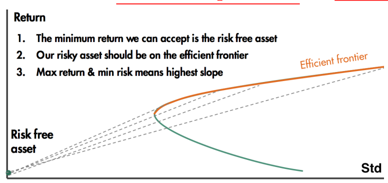


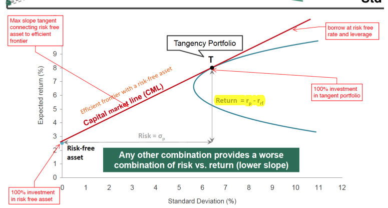


Def. (Risk-Free Asset) $\sigma_{rf} = 0$, $\text{Cov}(r_{rf}, r_i) = 0$, $\forall i$
Return = risk free rate (U.S. treasury bond of same duration)

Thm. Include more assets $\Rightarrow$ portfolio frontier improves: (moves toward upper-left: higher mean returns & lower risk)
($\because$)can always choose to ignore the new assets, so including them cannot make you worse off.
– Range of feasible solutions forms an area
– **Mean-Variance Frontier Portfolio:** portfolio that minimizes risk (measured by the std/variance), given an $\mathbb{E}$[return]
– **Portf Frontier:** locus of all frontier portfs in the mean-std plane
– **Efficient Frontier:** upper part of the portfolio frontier. Combination of risky assets with the best risk-return profile
– Convex curve ($\because$)mix 2 portfs (convex combo): stay inside frontier

Tangency Portf Best risky asset combination.
$\Rightarrow$ Has highest possible Sharpe Ratio
Capital Market Line, CML Combo of tangency portfolio + rf asset.
$\Rightarrow$ CML portfs have best possible SR: highest $\mathbb{E}$[return] for given risk
Sharpe Ratio Sharpe Ratio $= \frac{\mathbb{E}[\tilde{r_p}] - r_{RF}}{\sigma_p}$
$\Rightarrow$ **Select portfolio with best Sharpe ratio & desired risk**





Ex: (Minimum variance portfolio) 2 assets, find the minimum variance portfolio (set the derivative of portfolio's variance zero):
Weight: $w_1 = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}$, with $\sigma_{12} = \rho_{12}\sigma_1\sigma_2$.

Recap:
– Portf risk depends on cov, not stocks' individual σ_i
– Diversification can reduce some, but not all, risk
– CML has lowest risk for a given $\mathbb{E}$[return] & highest $\mathbb{E}$[return] for a given level of risk. All the portfs on CML have highest possible SR
– Pple are risk averse: should hold portfs on efficient frontier or CML
–Efficient Front=min-var front.Global MinVarPortf: leftmostOnFront

Risks and Return

Idiosyncratic risk no reward (diversifiable so no free lunch)
Systematic risk need reward:nobody wants to be exposed to it o'wise
Idea: If risk is completely diversifiable $\Rightarrow$ use risk-free rate r_{rf}.

Assume: Investors agree on the $\mathbb{E}$[return] of assets + Investors hold efficient frontier portfs + $\exists$a risk free asset r_{rf} at which investors can borrow & save + In equilibrium, demand = supply of assets
Then: –Every investor puts his money into two pots: the riskless asset + a single portfolio of risky assets: the Tangency Portfolio
– All investors hold the risky assets in same proportions: The weights in the Tangency Portfolio
⋆ **All investors hold Tangency Port $\Rightarrow$ Tgcy Port = Mkt Port**
Assume (CAPM):
– Investors like portfolios with high $\mathbb{E}[r_p]$, dislike portfolios with high σ_p, and don't care about anything else: rational + risk averse
– No asymmetric information: investors have same estimate of $E[r_i]$ and $\text{Cov}(r_i, r_j)$ $\forall$ risky assets
– $\exists$ a risk-free asset for both borrowing and investing.
Def. (Capital Asset Pricing Model, CAPM) Efficient portfolios are combinations of market portfolio and T-Bills. For asset i:

$$\mathbb{E}[r_i] = r_{rf} + \beta_i \times (\mathbb{E}[r_M] - r_{rf})$$

with $\beta_i = \frac{\text{Cov}(r_i, r_M)}{\text{Var}(r_M)} = \rho_{i,M}\frac{\sigma_i}{\sigma_M}$, r_M = Return of Market Portfolio.
For an arbitrary portfolio p:

$$\mathbb{E}[r_p] = r_{rf} + \beta_p \times (\mathbb{E}[r_M] - r_{rf})$$

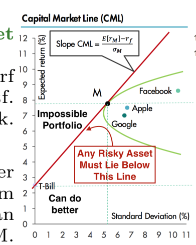
with $\beta_p = w_1\beta_1 + \ldots + w_n\beta_n$. **Note:** $SR(P) = \beta_p \frac{\sigma_m}{\sigma_p} SR(M)$

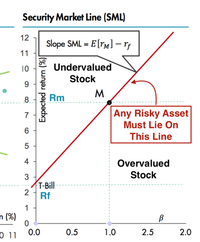
Corr If CAPM holds, then an asset's expected return depends on how it comoves with the market (β_i):

- $\beta_i = 1 \Rightarrow \mathbb{E}[r_i] = \mathbb{E}[r_m]$
- $\beta_i = 0 \Rightarrow \mathbb{E}[r_i] = r_{rf}$: only get compensated for systematic risk
- $\beta_i < 0 \Rightarrow \mathbb{E}[r_i] < r_{rf}$: need higher price (holding stock $\downarrow$ risk)

CAPM idea: Define: $r_{i,t+1} - r_{rf,t} = \alpha_i + \beta_i(r_{M,t+1} - r_{rf,t}) + \epsilon_{i,t+1}$
$\Longrightarrow$ $\mathbb{E}[\epsilon_{i,t+1}] = 0$, and $\text{Cov}(\epsilon_{i,t+1}, rM, t+1) = 0$
$\Longrightarrow$ $\beta_i = \beta_{im}$ measure i's systematic risk
$\Longrightarrow$ $\text{Var}(\epsilon_{i,t+1})$ measure i's idiosyncratic risk
$\Longrightarrow$ α_i measure i's return beyond its risk adjusted award (by CAPM)
$\Longrightarrow$ $\underbrace{\text{Var}(r_{i,t+1})}_{\text{total risk}} = \underbrace{\beta_i^2\text{Var}(r_{M,t+1})}_{\text{systematic risk}} + \underbrace{\text{Var}(\epsilon_{i,t+1})}_{\text{idiosyncratic risk}}$
α parameter: CAPM $\Longrightarrow$ no free reward $\Longrightarrow$ $\alpha_i = 0$
– If $\alpha_i > 0$: improve Sharpe Ratio by selling r_f bonds & buying i (can't be true in equilibrium: can't buy if nobody wants to sell)
– If you get $\alpha_i > 0$: check estimation errors. + Past value of α_i may not predict future values (CAPM theory is about expectations of future returns) + $\alpha_i > 0$ may be compensating for other risks.
CAPM $\Rightarrow$: Only syst risk rewarded + $\alpha = 0$
(β alone should explain all differences in $\mathbb{E}[r]$ among assets)
Def. (Risk Premium) $= E[r_M] - r_{rf} =$ reward for systematic risk
CAPM: r of project = $\mathbb{E}$[return] of an efficient portf with same syst risk

Capital & Security Market Line, CML & SML:
Efficient Portfs = combos of rf assets (T-Bills) & Market Portf. They do not have idiosyncratic risk.
CML = set of efficient portfs
Note: Stock below SML is over valued: a smaller risk premium is paid for a particular beta than would be expected from the CAPM.





By Zied Ben Chaouch

CAPM implementation: ⋆ $\mathbf{r_f}$ use T-Bill rates of similar duration
⋆ $\mathbf{r_m}$ use historical returns and future expectations (U.S.: $\sim 6-8\%$)
⋆$\beta_\mathbf{i}= \frac{\text{Cov}(r_i, r_m)}{\text{Var}(r_m)}$ OR lin.regress $\begin{cases} r_i - r_f = \alpha_i + \beta_i(r_m - r_f) + \epsilon \\ r_i = \alpha_i + \beta_i \cdot r_m + \epsilon \end{cases}$

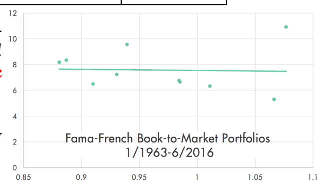
Note: Use Index as a proxy of the market (e.g., S&P 500)
+ Decide time horizon & frequency for the regression: use longest possible time horizon (with no structural break) & daily/ weekly freq
+ Regression: asset return ("y") vs. market return ("x"): β =slope

Asset Beta: β_A of the firm = weighted avg of **debt beta** (β_D) and **equity beta** (β_E): $\beta_A = \frac{D}{D+E}\beta_D + \frac{E}{D+E}\beta_E$ ($\sim$firms$\Rightarrow \sim \beta_\mathbf{A}$)

Fama French:: β not enough for asset pricing:
− Small stocks outperform large stocks
− Stocks with low mkt/book val ratio outperf stocks with high ratios

Book/MV decile	Avg. annual return	α
10^{th} decile	16.46%	4.95%
1^{st} decile	10.01%	−1.68%

Prop. CAPM dead: ∃ NO relationship between beta & returns!
⇒ **∃ more sources of systematic risk (exposure to market)**
-Small stocks outperform large
-Market-to-Book-Value ratio: low ratio stocks outperform large ratio



Thm. (Fama-French 3-Factor Model) Add 2 new sources of systematic risk to the CAPM: return on a portfolio that goes
−Size factor r_{smb}: long small stocks and short big stocks
−Value factor r_{hml}: long high B/M stocks and short low B/M stocks
Augmented SML (three-factor model):
$\mathbb{E}[r_i] - r_f = \beta_{im}\left(\mathbb{E}[r_m] - r_f\right) + \beta_{is}\mathbb{E}[r_{smb}] + \beta_{ih}\mathbb{E}[r_{hml}]$
⇒ $\alpha = 0$ + superior fit: fraction of var explained (R^2: 25% → 75%)

Thm. (APT) Extend CAPM: include more sources of aggregate risks

$$r_i - r_f = \alpha_i + \beta_{i1}\left(r_1^* - r_f\right) + \ldots + \beta_{in}\left(r_n^* - r_f\right) + \epsilon_i \quad \text{, where}$$

⋆ $r_1, \ldots, r_n$ = common risk factors (e.g., r_m)
⋆ $\beta_{i1}, \ldots, \beta_{in}$ = define asset i's exposure to each risk factor
⋆ ϵ_i = part of asset is risk unrelated to risk factors

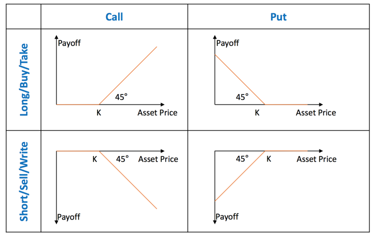
$$\mathbb{E}[r_i] - r_f = \beta_{i1}\left(\mathbb{E}[r_1^*] - r_f\right) + \ldots + \beta_{in}\left(\mathbb{E}[r_n^*] - r_f\right) \quad \text{, where}$$

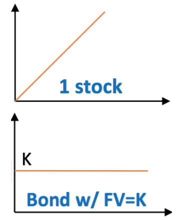
⋆ $\mathbb{E}[r_k^*] - r_f$ = premium on factor k
⋆ $\beta_{i1}, \ldots, \beta_{in}$ = asset i's loading on factor k
Need to Identify factors & estimate factor loadings+premiums of assets

Note: Reasonable descript° of return&risk, but don't give good factors
(APTvs.CAPM) **APT:** based on the factor model of returns&arbitrage
CAPM: based on investors' portf demand & equilibrium

Options:

Options = bet that stocks goes up or down.
+Hedge unwanted risk: bet on interest rates & foreign exchange
Call: Holder has right to **buy** an asset at the strike price on/by time T: bet that asset's stock price will go **up** on or by T
Put: Holder has right to **sell** an asset at the strike price on/by time T: bet that asset's stock price will go **down** on or by T
European: owner can exercise the option AT time T
American: owner can exercise the option on or before time T
Expiration/Maturity Date T. **Strike/Exercise Price** K.
Price of Underlying Asset S_t at time T.
Payoff Profiles:
In The Money: Option worth exercising: you will profit. The strike price K of **Call** (**Put**) option is **below** (**above**) the market price of the underlying asset.
(Out Of The Money) Option NOT worth exercising: you will NOT profit. The strike price K of **Call** (**Put**) option is **above** (**below**) the market price of the underlying asset.



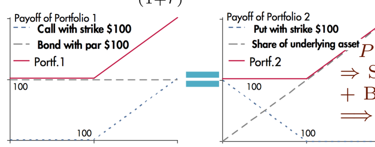


If Buy a European Call (price C) or Put (price P):
r = interest rate (EAR or (1+APR/n))

Option	Payoff	Profit
Call (buy)	$\max(S_T - K, 0)$	$\max(S_T - K, 0) - C(1+r)^T$
Call (sell)	$-\max(S_T - K, 0)$	$-\max(S_T - K, 0) + C(1+r)^T$
Put (buy)	$\max(K - S_T, 0)$	$\max(K - S_T, 0) - P(1+r)^T$
Put (sell)	$-\max(K - S_T, 0)$	$-\max(K - S_T, 0) + P(1+r)^T$

Note: Selling a put: potential loss may be infinite!

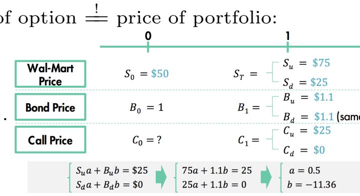
Thm. (Put-Call Parity)
$P + S = C + PV(K)$
$P + S = C + \frac{K}{(1+r)^T}$
(∵)No arbitrage
⇒ 2 portfs must have same cost
Note: Use r=APR



Note: If arbitrage:
$P + S - PV(K) = 5\$$ & $C = 4\$$
⇒ Sell 1P+1S & Buy 1 Bond
+ Buy 1C
⟹ make 1$!

Idea: Know price of bond+stock; Want price of option.
⇒ set up a portfolio investing a in stocks and b in bonds that exactly replicates the payoffs from the option

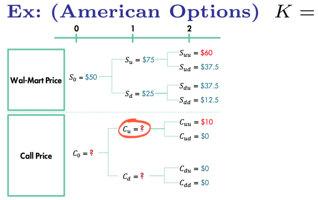
Prop. No arbitrage ⇒ the price of option $\stackrel{!}{=}$ price of portfolio:
$C_0 = aS_0 + bB_0$, where
$aS_u + bB_u = C_u$
$aS_d + bB_d = C_d$, with
$B_0 = 1\$$, $B_u = B_d = 1\$ \cdot (1+r)^T$.
Solution: $a = \frac{C_u - C_d}{S_u - S_d}$
$b = \frac{C_u S_d - C_d S_u}{B_u S_d - B_d S_u}$



Optn's hedge ratio/delta:#shares a needed to replicate a call option

American Optimal Strategy: $P_t = \max(S_t - K or K - S_t; aS_t + b)$
Optimal Strategy = max(option value if exercise, option value if don't)

Ex: (American Options) $K = 50\$$, $T = 2y$

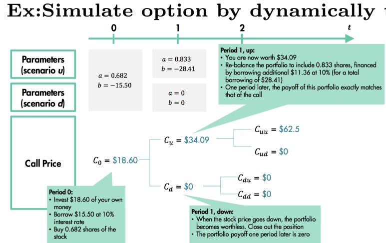


Replicating portfolio for $S_u = 75\$$:

$$\begin{matrix} 60a + 1.1b & = & 10 \\ 37.5a + 1.1b & = & 0 \end{matrix} \text{, with}$$

So: $a = 0.444$, $b = -15.15$:
EU: $C_u = 0.444 \cdot 60 - 15.15 = 18.18$
US: C_u=max{18.18,75-50}=25

Ex:Simulate option by dynamically trading stock/bond



Limits of Binomial: Trading takes place continuously + Price can take more than two possible values.
⇒ Let period-length→ 0: get BSM option pricing formula:
Thm. (Black-Scholes-Merton option pricing formula)
$C(S, K, T, \sigma) = S \cdot N(x) - K \cdot (1+r)^{-T} \cdot N(x - \sigma\sqrt{T})$, with
$x = \frac{\log\left(\frac{S}{K(1+r)^{-T}}\right)}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T}$ where:
⋆S = price of stock NOW, $N(.)$ = Gaussian CDF
⋆ r = annual riskless interest rate (units of a year).
⋆ σ = volatility of annual returns on underlying asset.

When ... increases	What happens to value of call	What happens to value of put	Explanation
Strike price (K)	Decreases	Increases	When the strike price is close to the asset price the call is closer to be in the money. Opposite for the put.
Price of underlying asset (S)	Increases	Decreases	Opposite logic from the above. When the asset price increases, the call option (which is a long position) increases, as expected.
Volatility of underlying asset (σ)	Increases	Increases	Remember, option holders bet on volatility. High σ means high volatility, so more value for the option holder.
Maturity (T)	Increases	Increases	With more time available, there is a higher chance the option will eventually be in the money.
Interest rate (r)	Increases	Decreases	The call price is given by $C = S + P - \frac{K}{(1+r)^T}$. So when r goes up, C goes down. Vice versa for a put option.

Def. (Implied Volatility) Given the BSM formula, find σ. The VIX index is a good indicator of investor's fear over the years.
if σ ⇑: good! get upside, not downside
Prop. BSM: Volatility increases the value of a Call Option:
−Things go really well vs. mildly well: Call worth a lot vs. a little!
−Things go really bad vs. mildly bad: Call worth nothing anyway
Prop. (Recipe for replicating payoffs with options) Start with the risk-free bond or asset + Adjust the downside (put) or downside (call) payoffs + Adjust the payoffs up (buy optn) or down (sell optn)

Corporate securities can be viewed as options:

Equity: A **call option** on firm's assets (K = its bond's redemption value): Equity = max(Assets − Debt, 0)

Debt: A portfolio combining the firm's **assets** + a short position in the **call** (with K = its bond's redemption value):
Debt = min(Assets, Debt) = Assets − max(Assets − Debt, 0)

Warrant: A **call** option on the firm's stock

Convertible Bond: A portfolio combining **straight bonds** + a **call** on the firms stock (with K related to the conversion ratio)

Callable bond: A portfolio combining **straight bonds** and a **call** written on the bonds

Ex: (Follow-up projects are an example of real options)
Assume: ⋆ Model-B decision has to be made in 1993.
⋆ Entry in 1993 with Model-A is prohibitively expensive
⋆ MC has the option to stop in 1993 (possible loss limited)
⋆ Investment needed for Model-B is $900M$ (twice that of A)
⋆ PV(operating profits from Model-B)= $464M$ in 1990
⋆ PV evolves with annual std of 39.5%; r_f = 6%.

Opportunity to invest in Model-B ≈ a 3y call option on asset worth $464M$ now (with $K = \$900M$)!
BSM: Value of Call = $55M$

Model	A	A+B
DCF	$-\$46M$	$-\$99M$
Option Value	$\$55M$	
Total Value	$\$9M$	$-\$99M$

Real options can change the investment decisions:
Naive DCF analysis tends to under-estimate value of strategic options:
⋆ Timing of projects is an option (American call)
⋆ Follow-on projects are options (American call)
⋆ Termination of projects are options (American put)
⋆ Expansion or contraction of production are optns (conversion optns).

Book Stuff

CFs

By now present value calculations should be a matter of routine. However, forecasting project cash flows will never be routine. Here is a checklist that will help you to avoid mistakes:

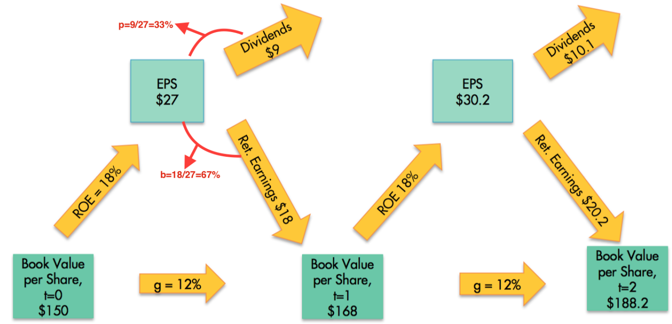
1. Discount cash flows, not profits.
   - Remember that depreciation is not a cash flow (though it may affect tax payments).
   - Concentrate on cash flows after taxes. Stay alert for differences between tax depreciation and depreciation used in reports to shareholders.
   - Exclude debt interest or the cost of repaying a loan from the project cash flows. This enables you to separate the investment from the financing decision.
   - Remember the investment in working capital. As sales increase, the firm may need to make additional investments in working capital, and as the project comes to an end, it will recover those investments.
   - Beware of allocated overhead charges for heat, light, and so on. These may not reflect the incremental costs of the project.
2. Estimate the project's incremental cash flows that is, the difference between the cash flows with the project and those without the project.
   - Include all indirect effects of the project, such as its impact on the sales of the firm's other products.
   - Forget sunk costs.
   - Include opportunity costs, such as the value of land that you would otherwise sell.
3. Treat inflation consistently.
   - If cash flows are forecasted in nominal terms, use a nominal discount rate.
   - Discount real cash flows at a real rate.
4. Separate investment and financing decisions by forecasting CFs as if the project is all equity financed.

Modigliani-Miller

- Modigliani and Millers (MMs) famous proposition 1 states that no combination is better than any other – that the firms overall market value (the value of all its securities) is independent of capital structure.
- Firms that borrow do offer investors a more complex menu of securities, but investors yawn in response. The menu is redundant. Any shift in capital structure can be duplicated or "undone" by investors. Why should they pay extra for borrowing indirectly (by holding shares in a levered firm) when they can borrow just as easily and cheaply on their own accounts?
- MM agree that borrowing raises the expected rate of return on shareholders' investments. But it also increases the risk of the firm's shares. MM show that the higher risk exactly offsets the increase in expected return, leaving stockholders no better or worse off.
- Proposition 1 is an extremely general result. It applies not just to the debt-equity trade- off but to any choice of financing instruments. For example, MM would say that the choice between long-term and short-term debt has no effect on firm value.
- The formal proofs of proposition 1 all depend on the assumption of perfect capital markets. MM's opponents, the "traditionalists", argue that market imperfections make personal borrowing excessively costly, risky, and inconvenient for some investors. This creates a natural clientele willing to pay a premium for shares of levered firms. The traditionalists say that firms should borrow to realize the premium.
- Proposition 1 is violated when financial managers find an untapped demand and satisfy it by issuing something new and different. The argument between MM and the traditionalists finally boils down to whether this is difficult or easy. We lean toward MM's view: Finding unsatisfied clienteles and designing exotic securities to meet their needs is a game that's fun to play but hard to win.
- If MM are right, the overall cost of capital – the expected rate of return on a portfolio of all the firm's outstanding securities – is the same regardless of the mix of securities issued to finance the firm. The overall cost of capital is usually called the company cost of capital or the weighted-average cost of capital (WACC). MM say that WACC doesn't depend on capital structure. But MM assume away lots of complications. The first complication is taxes. When we recognize that debt interest is tax-deductible, and compute WACC with the after-tax interest rate, WACC declines as the debt ratio increases. There is more – lots more – on taxes and other complications in the next two chapters.

Stocks:

ex:



Recitation Examples

Risk and Return:

Ex:

Expected return & β of security A + market portfolio.
CAPM holds: **find risk-free rate**

Security	$\mathbb{E}[\mathbf{r}]$	β
A	21%	1.6
Market	15%	1.0

CAPM $\Rightarrow r_A = r_f + \beta_A \cdot (r_m - r_f) \Rightarrow 21\% = r_f + 1.6 \cdot (15\% - r_f)$
$\Rightarrow \mathbf{r_f = 5\%}$

Ex: $\beta_{ABC} = 0.4$, $r_f = 5\%$, market risk premium $= 6\%$.

- Find $\mathbb{E}[r_{ABC}]$: CAPM $\Rightarrow \mathbb{E}[r_{ABC}] = 5\% + 0.4\% \cdot 6\% = 7.4\%$
- $\sigma_m = 15\%$, $\rho(ABC, m) = 0.3$. Find σ_{ABC}.
  $\rho(ABC, m) = \frac{\text{Cov}(ABC,m}{\sigma_{ABC}\sigma_m}$, $\text{Cov}(ABC, m) = \beta_{ABC}\sigma_m^2$
  $\Rightarrow \sigma_{ABC} = \beta_{ABC}\frac{\sigma_m}{\rho(ABC,m)} = 20\%$
- Expect $Price(ABC) = 150\$$ in 1yr. Find price today (p).
  $\mathbb{E}[r_{ABC}] = \frac{150\$ - p}{p} = 7.4\% \Rightarrow p = \frac{150\$}{1+7.4\%} = 139.66\$$

Ex: Using the properties of CML & SML, are the following scenarios are (in)consistent with CAPM ?

Security	$\mathbb{E}[\mathbf{r}]$	β
A	25%	0.8
B	15%	1.2

Inconsistent: $\beta \uparrow \Rightarrow \uparrow \mathbb{E}[r]$
Not inconsistent if historical data

Security	$\mathbb{E}[\mathbf{r}]$	$\sigma(\mathbf{r})$
A	25%	30%
M	15%	30%

Inconsistent: A lies above CML
$\Rightarrow$ inefficient market portfolio
$SR_m = \frac{\mathbb{E}[r_m]-r_f}{\sigma_m} < \frac{\mathbb{E}[r_A]-r_f}{\sigma_A} = SR_A$

Security	$\mathbb{E}[\mathbf{r}]$	$\sigma(\mathbf{r})$
A	25%	55%
M	15%	30%
F	5%	0%

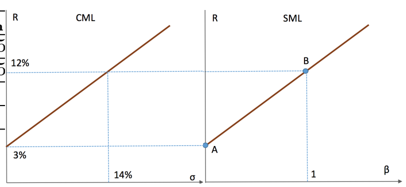
Inconsistent: A lies above CML
$\Rightarrow$ inefficient market portfolio
$SR_m = 0.36 < 0.33 = SR_A$

Security	$\mathbb{E}[\mathbf{r}]$	β
A	20%	1.5
M	15%	1.0
F	5%	0

Consistent: A lies on SML
$r_f + \beta_A \cdot (r_m - r_f) = r_A$ ✓

Ex:

Stock	$\mathbf{r_i}$	$\sigma_\mathbf{i}$	$\rho_\mathbf{im}$
1	6.825 %	17%	0.35
2	22.125 %	35%	0.85
M	12%	14%	1
F	3%	0%	0



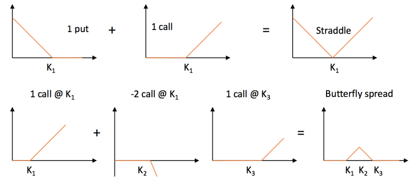
1) Find $\mathbb{E}[r_1]$ & $\mathbb{E}[r_2]$:
$\beta_{1m} = \rho_{1m}\frac{\sigma_1}{\sigma_m} = 0.425$
$\beta_{2m} = \rho_{2m}\frac{\sigma_2}{\sigma_m} = 2.125$
$-$CAPM Eq: $r_i = r_f + \beta_{im}(r_m - r_f) \Rightarrow r_1 = 6.825\%$ & $r_1 = 22.125\%$
$-$Geometry: SML line $r_i = 9\beta_{im} + 3 \Rightarrow r_1 = 6.825\%$ & $r_1 = 22.125\%$

Intuition:
$\beta_{1m} = 0.425 \Rightarrow$ stock risk prem. should be half of market risk prem.
$\beta_{2m} = 2.125 \Rightarrow$ stock risk prem. should be double of market risk prem.

2) Given $\rho_{12} = 0.85$, $w_1 = 15\%$ & $w_2 = 85\%$:
$\mathbb{E}[r_p] = w_1\mathbb{E}[r_1] + w_2\mathbb{E}[r_2] = 19.84\%$
$\sigma(r_p) = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{12}\sigma_1\sigma_2} = 31.95\%$

3) Construct portfolio with same return as r_p & lower risk
$0.1984 = \mathbb{E}[r_p] = w_1\mathbb{E}[r_m] + w_2\mathbb{E}[r_f] = 0.12w_1 + 0.03(1 - w_1)$
$\Rightarrow w_1 = 187\%$ $w_2 = -87\%$ (borrow at rf rate to lever up)
$\sigma_{new} = w_1\sigma_m = 26.18\%$ OR use geometry (CML):
$r = a\sigma + b \Rightarrow r_i = 0.643\sigma_i + 3 \stackrel{!}{=} 19.84\% \Rightarrow \sigma_{new} = 26.18\%$

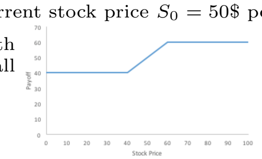
Options:



Ex: (Payoff Profiles)
Area of curve: large+extends far from strike price on both directions $\Longrightarrow$ bet on volatility.
O'wise: expect price to move within a short range around strike price.

Let $T = 6-$month maturity & current stock price $S_0 = 50\$$ per share.

(a) Buy 1 share + buy 1 put with exercise price $K = 40\$$ + sell 1 call with exercise price $K = 60\$$.



(b) Buy 1 put + 1 call with exercise price $K = 50\$$ + sell 1 put with exercise price $K = 40\$$ + sell 1 call with exercise price $K = 60\$$.

Ex: (Real Options)
You financed a new movie for $PV = \$75M$. You can do a sequel which would cost $\$100M$ to produce and could be made any time in the next five years.

1. If we think of the possibility of making a sequel as an option, is it a call or a put? European or American?
   Solution: *American call*
2. Strike price & time to expiration/maturity?
   Solution: $K = \$100M, T = 5y$

   Sequel will produce 15% less CFs than 1st movie produces (e.g. if 1st produces $\$100M$, 2nd will produce $\$85M$).
3. Find minimum free CF (excluding the initial investment) generated by 1st movie for which you'd exercise the option and make the sequel?
   Solution: *You would exercise the option when 1st movie is expected to make more money than it costs to produce:*
   $0.85 \cdot CF \geq \$100M \implies CF \geq 100M/0.85 = \$117.65M$